RIP Analysis of the Measurement Matrix for Compressive Sensing-Based MIMO Radars

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Overview

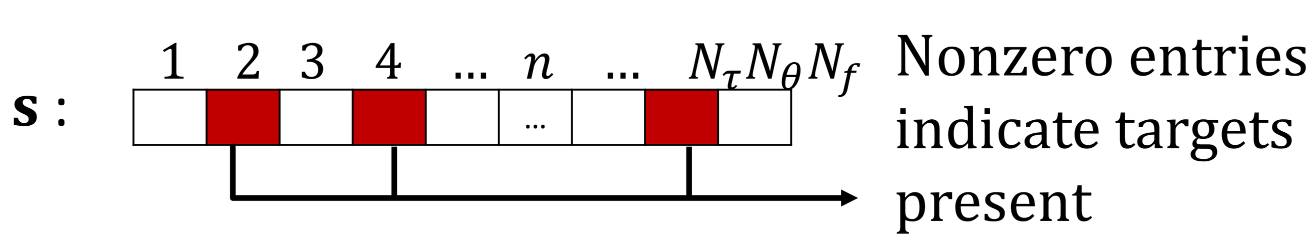
- This paper considers range-angle-Doppler estimation in collocated, compressive sensing-based MIMO (CS-MIMO) radars with arbitrary array configuration.
- In the literature, the effectiveness of CS-MIMO radars has been studied mostly via simulations. Existing theoretical results for MIMO radars with linear arrays cannot be easily extended to arbitrary array configurations.
- This paper analyzes the restricted isometry property (RIP) of the measurement matrix.
- A scheme is proposed that selects the subset of receive antennas with the smallest cardinality that meet the RIP conditions.

System Model

Consider the collocated MIMO radar system of [1, Section II]. The range-angle-Doppler space is discretized on the grid $\mathcal{T} \times \Theta \times \mathcal{D}$ with $|\mathcal{T}| = N_\tau, |\Theta| = N_\theta, |\mathcal{D}| = N_f$. The M_t transmitted waveforms are Gaussian signals with variance $\sigma_0^2 = 1/L$. The fusion center collects the samples from all M_r receivers and P pulses and stacks them into vector $\mathbf{z} \in \mathbb{C}^{LPM_r}$. The model obeys

$$\mathbf{z} = \mathbf{\Psi s} + \mathbf{n}, \qquad (1)$$

where $\mathbf{s} \in \mathbb{C}^{N_\tau N_\theta N_f}$ denotes the sparse target vector



$\mathbf{\Psi} \in \mathbb{C}^{(LPM_r)\times(N_\tau N_\theta N_f)}$ is the measurement matrix with n-th column $\mathbf{\Psi}_n = \mathbf{v}_r(\theta_n) \otimes \{\mathbf{D}(f_n) \otimes [\mathbf{X}_{\tau_n}\mathbf{v}_t(\theta_n)]\}$.

- $\mathbf{v}_{r/t}(\theta_n)$ receive/transmit steering vector
- $\mathbf{D}(f_n)$ Doppler vector $\left[1, e^{j2\pi f_n T_p}, \dots, e^{j2\pi f_n T_p\,(P-1)}\right]^T$
- $\mathbf{X}_{\tau_n}$ waveform matrix $[\mathbf{x}_{1,\tau_n}, \dots, \mathbf{x}_{M_t,\tau_n}]$
- $\mathbf{x}_{m,\tau_n}$ the m-th waveform shifted by τ_n

θ_n, τ_n, f_n denote angle, delay and Doppler frequency associated with the n-th grid point.

The restricted isometry property (RIP) of $\mathbf{\Psi}$ plays an important role in guaranteeing the recoverability and estimation performance of $\mathbf{s}$.

Observations on the Gram of Normalized $\mathbf{\Psi}$

The Gram of normalized $\mathbf{\Psi}$ equals $\mathbf{G} = \frac{\mathbf{\Psi}^H\mathbf{\Psi}}{M_t M_r P}$ with $\mathbf{G}_{nl} = \frac{\langle \mathbf{\Psi}_n, \mathbf{\Psi}_l\rangle}{M_t M_r P}$ where
$\langle \mathbf{\Psi}_n, \mathbf{\Psi}_l\rangle = \langle \mathbf{v}_r(\theta_n), \mathbf{v}_r(\theta_n)\rangle\langle \mathbf{D}(f_n), \mathbf{D}(f_l)\rangle\langle \mathbf{X}_{\tau_n}\mathbf{v}_t(\theta_n), \mathbf{X}_{\tau_l}\mathbf{v}_t(\theta_l)\rangle$.

To bound the entries of $\mathbf{G}$, we have four cases

- **Case (i)**: for the diagonal entries, i.e., $n = l$,
$$\Pr\left(\left|\frac{\|\mathbf{\Psi}_n\|_2^2}{M_t M_r P} - 1\right| > t\right) \le 2\exp\left(-\frac{Lt^2}{16}\right)$$
- **Case (ii)**: for $\tau_n \ne \tau_l$, we have
$$\Pr\left(\left|\frac{\langle \mathbf{\Psi}_n, \mathbf{\Psi}_l\rangle}{M_t M_r P}\right| > t\right) \le 4\exp\left(-\frac{Lt^2}{4+4t}\right)$$
- **Case (iii)**: for $\tau_n = \tau_l, \theta_n \ne \theta_l$, we have
$$\Pr\left(\left|\frac{\langle \mathbf{\Psi}_n, \mathbf{\Psi}_l\rangle}{M_t M_r P}\right| > t\right) \le 4\exp\left(-\frac{Lt^2}{4C_1 + 2C_2 t}\right)$$
where C_1 and C_2 are constants, which holds if
$$M_t M_r \ge 2/t\,\phi_{\theta_n,\theta_l}(M_r)\phi_{\theta_n,\theta_l}(M_t).$$
- **Case (iv)**: for $\tau_n = \tau_l, \theta_n = \theta_l, f_n \ne f_l$, we have
$$\Pr\left(\left|\frac{\langle \mathbf{\Psi}_n, \mathbf{\Psi}_l\rangle}{M_t M_r P}\right| > t\right) \le \exp\left(-\frac{Lt^2}{10}\right)$$

as long as $P \ge \sqrt{2}(1/t + 1)\phi_{f_n,f_l}(P)$.

In the above,

$$\phi_{\theta_n,\theta_l}(M_o) \equiv |\langle \mathbf{v}_o(\theta_n), \mathbf{v}_o(\theta_l)\rangle|, o \in \{r, t\},$$
$$\phi_{f_n,f_l}(P) \equiv |\langle \mathbf{D}(f_n), \mathbf{D}(f_l)\rangle|.$$

Based on these bounds the following theorem holds.

Measurement Matrix Satisfying The RIP

Theorem 1: Let $\widetilde{\mathbf{\Psi}} \equiv \mathbf{\Psi}/\sqrt{M_t M_r P}$. For any $\delta_K \in (0,1)$, there exist C_1 such that $\widetilde{\mathbf{\Psi}}$ satisfies RIP of order K with parameter δ_K with probability exceeding $1 - 4(N_\tau N_\theta N_f)^{-1}$ whenever

$$L \ge C_0 \delta_K^{-2} K^2 \log(N_\tau N_\theta N_f), \qquad (2a)$$
$$M_t M_r \ge 2\delta_K^{-1} K \beta_\Theta(M_t, M_r), \qquad (2b)$$
$$P \ge \sqrt{2}(\delta_K^{-1}K + 1)\beta_D(P), \qquad (2c)$$

where $\beta_\Theta(M_t, M_r) \equiv \sup_{\theta_n,\theta_l \in \Theta, n \ne l} \phi_{\theta_n,\theta_l}(M_t)\phi_{\theta_n,\theta_l}(M_r)$ and

$$\beta_D(P) \equiv \sup_{f_n,f_l \in \mathcal{D}, n \ne l} \phi_{f_n,f_l}(P).$$

Remark: Theorem 1 characterizes the RIP of normalized $\mathbf{\Psi}$ under the conditions of (2) for arbitrary array configuration and grid set $\mathcal{T} \times \Theta \times \mathcal{D}$.

Nodes Selection

In the CS-MIMO radar literature, the virtual ULA MIMO radars in [2] require that $M_t M_r$ equals the cardinality of Θ. Random linear array MIMO radars require that $M_t M_r \propto K^2 \log^2 N_\theta$ [3].

In this paper, we propose a scheme to minimize the number of selected receive nodes w.r.t the nodes' positions, under the condition of (2b).

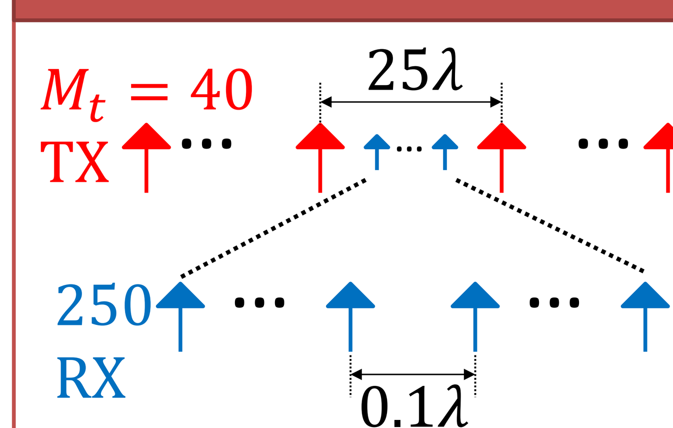
Given a very dense array of M receive nodes at positions $\tilde{\mathbf{y}}$, we assign a weight $w_m \in [0,1]$ and solve

$$(P1) \quad \min_{\mathbf{w}} \mathbf{1}^T\mathbf{w} \equiv [1, \dots, 1][w_1, \dots, w_M]^T$$
$$s.t. \sup_{\theta_n,\theta_l \in \Theta} \phi_{\theta_n,\theta_l}(M_t) f(\mathbf{w}) \le \frac{M_t \delta_K}{2K}\mathbf{1}^T\mathbf{w}$$
$$\mathbf{1}^T\mathbf{w} \ge 4, 0 \le w_m \le 1, m \in \mathbb{N}_M^+$$

A suboptimal sensor selection set can be generated from the solution of (25), by taking its M_r largest entries.

Remark: It is similar to optimize w.r.t the transmit array given a fixed receive array.

Numerical Example



The receive nodes are chosen from 0.1λ-spaced ULA with 250 nodes. We consider $K = 3$ targets in the angular region $\sin\theta \in [0, 0.15]$ discretized uniformly with interval $\Delta_{\sin\theta} = 0.001$.

Figure 1 shows the result by choosing the $M_r = 30$ largest entries of the solution of (P1) while (2b) is satisfied.

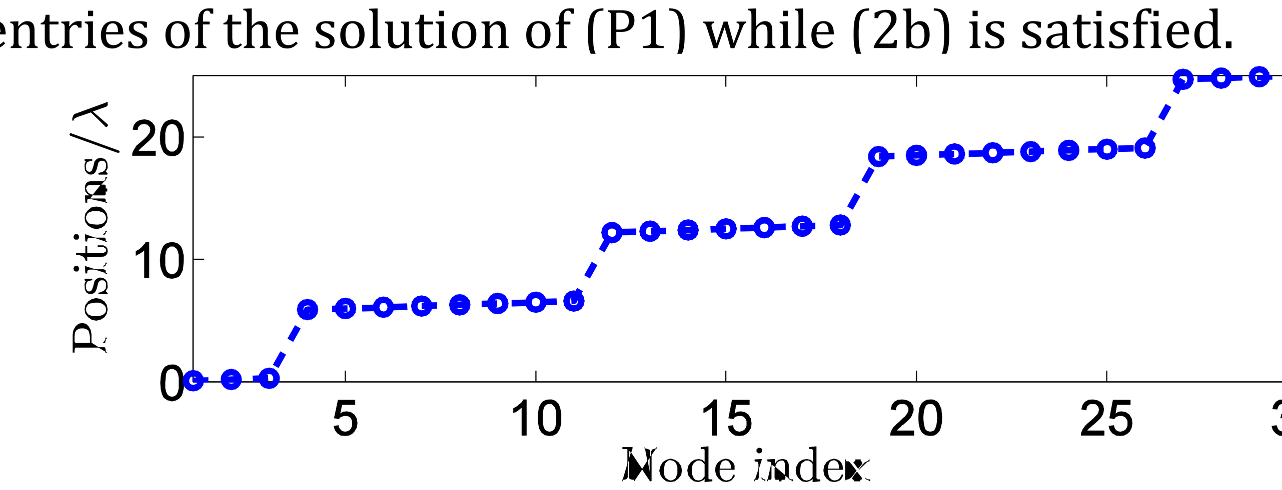


Fig. 1. Positions of the 30 selected nodes by the proposed scheme.

For comparison, the virtual array setup in [2] requires $M_r = 50$. The random array in [3] requires $M_t M_r \ge 2121$. It is clear that our method produces CS-MIMO radars with the fewest nodes.

References

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