

RIP Analysis of the Measurement Matrix for Compressive Sensing-Based MIMO Radars

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Abstract—This paper considers range-angle-Doppler estimation in collocated, compressive sensing-based MIMO (CS-MIMO) radars with arbitrary array configuration. In the literature, the effectiveness of CS-MIMO radars has been studied mostly via simulations. Although there exist some theoretical results for MIMO radars with linear arrays, those cannot be easily extended to arbitrary array configurations. This paper analyzes the restricted isometry property (RIP) of the measurement matrix. The RIP conditions involve, among other quantities, the number of transmit and receive antennas. A scheme is proposed that selects the subset of receive antennas with the smallest cardinality that meet the RIP conditions.

Index Terms—Collocated MIMO radar, sparse sensing, restricted isometry property, antenna selection.

I. INTRODUCTION

Recently, compressive sensing (CS) [1] based MIMO radars were shown to achieve the superior resolution of collocated MIMO radars with significantly fewer measurements [2]–[4]. If there is a small number of targets in the target space, target estimation can be formulated as a sparse signal recovery problem. The work in [5] provided the first nonuniform recovery guarantee for range-angle-Doppler estimation and the corresponding bounds on the number of transmit/receive antennas and measurements. However, the results only apply to CS-MIMO radars with virtual uniform linear array (ULA) configuration, i.e., M_r -element $\lambda/2$ -spaced receive array and M_t -element $M_r\lambda/2$ -spaced transmit array. Also, in [5], the angular space has to be discretized on a uniform grid with spacing $\frac{2}{M_t M_r}$. The extension of the results of [5] to general array configurations is nontrivial. Spatial CS for MIMO radars with random transmit/receive array was proposed in [6], [7] for angle estimation. A nonuniform recovery guarantee was provided in [6], [7] based on the isotropy property of the measurement matrix. The work in [7] also provided a uniform recovery guarantee based on the coherence analysis of the measurement matrix. However, the analysis cannot be extended to the range-angle-Doppler estimation.

In this paper, we consider the range-angle-Doppler estimation using CS-based collocated MIMO radars with arbitrary array configuration. Our goal is to provide the restricted isometry property (RIP) of the measurement matrix, which can then be readily used to derive uniform recovery guarantees. Towards this goal, we derive a unified upper bound on the

entries of the Gram of the measurement matrix. To relate this with the RIP, we adopt the well-known scheme in [9] based on *Geršgorin's Disc Theorem*, which was originally applied for the RIP of Toeplitz matrices. The RIP conditions involve the number and positions of the antennas. Based on this observation, we propose a scheme that selects the subset of receive antennas with the smallest cardinality that meet the RIP conditions.

The paper is organized as follows: Section II introduces the sparse model for collocated MIMO radar system. In Section III, we present the RIP analysis of the measurement matrix. Also, we propose an optimization scheme to minimize the required number of receive antennas, which is validated in Section IV. Conclusions are presented in Section V.

II. SIGNAL MODEL

Consider the collocated MIMO radar system of [4, Section II] equipped with transmit and receive arrays with M_t and M_r antennas, respectively. Let us assume that there are K moving targets and that the environment is clutter free. We are interested in target parameters including the time delay from the transmitter to the receiver via the k -th target, i.e., τ_k , the target azimuth angle, θ_k , and Doppler frequency, f_k , for all $k \in \mathbb{N}_K^+$. It holds that $\tau_k = 2d_k/v_c$ and $f_k = 2v_k f_c/v_c$, where d_k, v_k, f_c and v_c denote target range, target radial velocity, carrier frequency and speed of light, respectively. Without loss of generality, we use delay instead of range. To exploit the target space sparsity, the delay-angle-Doppler space is discretized on the grid $\mathcal{T} \times \Theta \times \mathcal{D}$ with $|\mathcal{T}| = N_\tau, |\Theta| = N_\theta$, and $|\mathcal{D}| = N_f$. All grid points are ordered and labeled by the index set $\mathcal{I} \triangleq \{1, \dots, N_\tau N_\theta N_f\}$. It is assumed that the targets fall on grid points.

The transmit array emits P probing pulses with pulse repetition interval T_{PRI} . Each receiver obtains L T_s -spaced samples from the target returns during each pulse. The fusion center collects the samples from all receivers and stacks them into vector $\mathbf{z} \in \mathbb{C}^{LP M_r}$. From [4], the model obeys

$$\mathbf{z} = \Psi \mathbf{s} + \mathbf{n}, \quad (1)$$

where \mathbf{n} is the interference/noise vector, $\mathbf{s} \in \mathbb{C}^{N_\tau N_\theta N_f}$ denotes a sparse target vector whose K nonzero entries correspond to the complex reflection coefficients of the targets, and $\Psi \in \mathbb{C}^{(LP M_r) \times (N_\tau N_\theta N_f)}$ is the measurement matrix; its n -th column is associated with the n -th grid point as follows

$$\Psi_n = \mathbf{v}_r(\theta_n) \otimes \{\mathbf{D}(f_n) \otimes [\mathbf{X}_{\tau_n} \mathbf{v}_t(\theta_n)]\}, \forall n \in \mathcal{I}, \quad (2)$$

where \otimes is the Kronecker product, $\mathbf{v}_r(\theta) \in \mathbb{C}^{M_r}$ is the receive steering vector defined as

$$\mathbf{v}_r(\theta) \triangleq \left[e^{j2\pi(\mathbf{d}_1^r, \mathbf{w}(\theta))/\lambda}, \dots, e^{j2\pi(\mathbf{d}_{M_r}^r, \mathbf{w}(\theta))/\lambda} \right]^T, \quad (3)$$

($\mathbf{v}_t(\theta)$ is the transmit steering vector and is respectively defined) with $\mathbf{d}_m^r \triangleq [x_m^r, y_m^r]^T$ denoting the two-dimensional coordinates of the m -th receive antenna, $\mathbf{w}(\theta) \triangleq [\cos(\theta) \sin(\theta)]^T$, and

$$\begin{aligned} \mathbf{D}(f) &\triangleq \left[1, e^{j2\pi f T_{PRI}}, \dots, e^{j2\pi f T_{PRI}(P-1)} \right]^T, \\ \mathbf{X}_\tau &\triangleq [\mathbf{x}_{1,\tau}, \dots, \mathbf{x}_{M_t,\tau}], \\ \mathbf{x}_{m,\tau} &\triangleq [x_m[\tau], \dots, x_m[(L-1)T_s + \tau]]^T, \quad m \in \mathbb{N}_{M_t}^+ \end{aligned} \quad (4)$$

with λ and $x_m[t]$ denote, respectively, the carrier wavelength and the sample of the m -th transmit waveform at time index t . We assume that transmit waveforms are jointly Gaussian with zero mean and variance $\sigma_0^2 = 1/L$. We assume that the targets are moving slowly, thus the Doppler effect can be approximated as constant during one pulse.

The estimation of the target parameters can be achieved by various sparse recovery algorithms, including the ℓ_1 minimization algorithms, or greedy algorithms. It is well-known that the restricted isometry property (RIP) [1] of the measurement matrix Ψ plays an important role on guaranteeing the recoverability and estimation performance of \mathbf{s} . In order to provide the performance of CS-MIMO radars, it is essential to characterize the RIP of Ψ .

III. MAIN RESULTS

In this section, we analyze the RIP of Ψ . Ahead of the RIP analysis, we provide some observations on the Gram of matrix Ψ . Let us first state one lemma which will be used later.

Lemma 1 (Lemma 5 in [9]): Let $\mathbf{x} \in \mathbb{C}^N$ and $\mathbf{y} \in \mathbb{C}^N$ be vectors with i.i.d complex Gaussian entries with zero mean and variance σ^2 . Then for every $t > 0$ it holds that

$$\Pr(\|\mathbf{x}\|_2^2 - \mathbb{E}\{\|\mathbf{x}\|_2^2\} \geq t) \leq e^{-\frac{t^2}{16N\sigma^4}}, \quad (5a)$$

$$\Pr(\|\mathbf{x}\|_2^2 - \mathbb{E}\{\|\mathbf{x}\|_2^2\} \leq -t) \leq 2e^{-\frac{t^2}{16N\sigma^4}}, \quad (5b)$$

$$\Pr(|\langle \mathbf{x}, \mathbf{y} \rangle| \geq t) \leq 2e^{-\frac{t^2}{4\sigma^2(N\sigma^2+t/2)}}. \quad (5c)$$

where $\langle \mathbf{x}, \mathbf{y} \rangle \triangleq \mathbf{x}^H \mathbf{y}$, and $(\cdot)^H$ denotes Hermitian transpose.

A. Observations on The Gram of The Normalized Ψ

Note that $\mathbb{E}\{\|\Psi_n\|_2^2\} = M_t M_r P$. Since in the compressive sensing literature measurement matrices with normalized columns are typically considered, we will find bounds for the diagonal and off-diagonal entries of $\mathbf{G} \triangleq \frac{\Psi^H \Psi}{M_t M_r P}$, i.e., $\frac{\langle \Psi_n, \Psi_l \rangle}{M_t M_r P}$ for all $n, l \in \mathcal{I}$, where the inner product of two columns of Ψ is given by

$$\begin{aligned} \langle \Psi_n, \Psi_l \rangle &= \langle \mathbf{v}_r(\theta_n), \mathbf{v}_r(\theta_l) \rangle \langle \mathbf{D}(f_n), \mathbf{D}(f_l) \rangle \\ &\quad \times \langle \mathbf{X}_{\tau_n} \mathbf{v}_t(\theta_n), \mathbf{X}_{\tau_l} \mathbf{v}_t(\theta_l) \rangle. \end{aligned} \quad (6)$$

When $(\tau_n, \theta_n, f_n) = (\tau_l, \theta_l, f_l)$, the inner product becomes the square of the norm, i.e., $\|\Psi_n\|_2^2 = M_r P \|\mathbf{X}_{\tau_n} \mathbf{v}_t(\theta_n)\|_2^2$.

For all the entries, the following four cases are considered:

Case (i) $n = l$: In this case, we only need to consider $\|\Psi_n\|_2^2$ for any $n \in \mathcal{I}$. Denote by $\mathbf{g} \in \mathbb{C}^L$ the product $\mathbf{X}_{\tau_n} \mathbf{v}_t(\theta_n)$. The i -th entry of \mathbf{g} is given by $g_i = [x_1[(i-1)T_s + \tau_n], \dots, x_{M_t}[(i-1)T_s + \tau_n]] \mathbf{v}_t(\theta_n)$, which is a weighted sum of M_t i.i.d jointly Gaussian random variables of variance $1/L$. Therefore, the entries of \mathbf{g} are independent identical distributed according to $\mathcal{CN}(0, M_t/L)$. Based on (5b) in Lemma 1, we get

$$\begin{aligned} \Pr(\|\Psi_n\|_2^2 - \mathbb{E}\{\|\Psi_n\|_2^2\} \geq M_r P t) \\ = \Pr(\|\mathbf{g}\|_2^2 - \mathbb{E}\{\|\mathbf{g}\|_2^2\} \geq t) \leq 2e^{-\frac{L t^2}{16 M_t^2}}. \end{aligned} \quad (7)$$

Substituting $\mathbb{E}\{\|\mathbf{g}\|_2^2\} = M_t$ and $t \equiv M_t t$ into (7), we get

$$\Pr\left(\left|\frac{\|\Psi_n\|_2^2}{M_t M_r P} - 1\right| \geq t\right) \leq 2e^{-\frac{L t^2}{16}}. \quad (8)$$

Case (ii) $\tau_n \neq \tau_l$: We know that $\mathbf{X}_{\tau_n} \mathbf{v}_t(\theta_n)$ has i.i.d complex Gaussian entries with zero mean and variance M_t/L ; the same holds for $\mathbf{X}_{\tau_l} \mathbf{v}_t(\theta_l)$. However, the sum terms in $\langle \mathbf{X}_{\tau_n} \mathbf{v}_t(\theta_n), \mathbf{X}_{\tau_l} \mathbf{v}_t(\theta_l) \rangle$ are no longer mutually independent. Following the splitting trick of [5], [8], [9], we can split the terms into two equal-sized groups, each of which only contains mutually independent terms. Applying (5c) in Lemma 1 to both groups of sums and using Boole's inequality, we obtain

$$\Pr(|\langle \mathbf{X}_{\tau_n} \mathbf{v}_t(\theta_n), \mathbf{X}_{\tau_l} \mathbf{v}_t(\theta_l) \rangle| \geq 2t) \leq 4e^{-L \frac{t^2}{2M_t^2 + 2M_t t}}.$$

Combining with (6), we get

$$\begin{aligned} \Pr(|\langle \Psi_n, \Psi_l \rangle| \geq 2M_r P t) \\ \leq \Pr(|\langle \Psi_n, \Psi_l \rangle| \geq 2t \phi_{\theta_n, \theta_l}(M_r) \phi_{f_n, f_l}(P)) \\ \leq 4e^{-L \frac{t^2}{2M_t^2 + 2M_t t}}, \end{aligned} \quad (9)$$

where

$$\begin{aligned} \phi_{\theta_n, \theta_l}(M_r) &\triangleq |\langle \mathbf{v}_r(\theta_n), \mathbf{v}_r(\theta_l) \rangle| \in [0, M_r], \\ \phi_{f_n, f_l}(P) &\triangleq |\langle \mathbf{D}(f_n), \mathbf{D}(f_l) \rangle| \in [0, P]. \end{aligned} \quad (10)$$

Substituting t in (9) by $M_t t/2$, we get

$$\Pr\left(\left|\frac{\langle \Psi_n, \Psi_l \rangle}{M_t M_r P}\right| \geq t\right) \leq 4e^{-L \frac{t^2}{4+4t}}. \quad (11)$$

Case(iii) $\tau_n = \tau_l, \theta_n \neq \theta_l$: We need to find the bound on $|\langle \mathbf{X}_{\tau_n} \mathbf{v}_t(\theta_n), \mathbf{X}_{\tau_n} \mathbf{v}_t(\theta_l) \rangle|$. According to [5, Lemma 11], we have

$$\Pr\left(\underbrace{|\langle \mathbf{X}_{\tau_n} \mathbf{v}_t(\theta_n), \mathbf{X}_{\tau_n} \mathbf{v}_t(\theta_l) \rangle - \langle \mathbf{v}_t(\theta_n), \mathbf{v}_t(\theta_l) \rangle|}_{\triangleq \chi} \geq M_t t\right) \quad (12)$$

$$\triangleq \mathcal{P}_\chi \leq 2e^{-L \frac{t^2}{C_1 + C_2 t}},$$

where $C_1 \approx 2.50$ and $C_2 \approx 7.69$. It is also clear that

$$\begin{aligned} \mathcal{P}_\chi &= \Pr\left(\underbrace{M_r P |\chi| + \phi_{\theta_n, \theta_l}(M_r) \phi_{\theta_n, \theta_l}(M_t) P}_{\triangleq A} \geq \zeta\right) \\ &\geq \Pr\left(\underbrace{|\langle \mathbf{v}_r(\theta_n), \mathbf{v}_r(\theta_l) \rangle \langle \mathbf{D}(f_n), \mathbf{D}(f_l) \rangle (\chi + \langle \mathbf{v}_t(\theta_n), \mathbf{v}_t(\theta_l) \rangle)|}_{\triangleq B} \geq \zeta\right) \\ &= \Pr(|\langle \Psi_n, \Psi_l \rangle| \geq \zeta), \end{aligned} \quad (13)$$

where $\zeta \triangleq M_t M_r P t + \phi_{\theta_n, \theta_l}(M_r) \phi_{\theta_n, \theta_l}(M_t) P$ and

$$\phi_{\theta_n, \theta_l}(M_t) \triangleq |\langle \mathbf{v}_t(\theta_n), \mathbf{v}_t(\theta_l) \rangle| \in [0, M_t] \quad (14)$$

and the second inequality holds because

$$A \geq |\langle \mathbf{v}_r(\theta_n), \mathbf{v}_r(\theta_l) \rangle \langle \mathbf{D}(f_n), \mathbf{D}(f_l) \rangle| \times (|\chi| + |\langle \mathbf{v}_t(\theta_n), \mathbf{v}_t(\theta_l) \rangle|) \geq B.$$

If $M_r M_t t \geq \phi_{\theta_n, \theta_l}(M_r) \phi_{\theta_n, \theta_l}(M_t)$, it holds that $2M_t M_r P t \geq \zeta$. Now, the bound on the inner product can be written as

$$\Pr(|\langle \Psi_n, \Psi_l \rangle| \geq 2M_t M_r P t) \leq \mathcal{P}_\chi \quad (15)$$

or, equivalently, if t is substituted by $t/2$,

$$\Pr\left(\left|\frac{\langle \Psi_n, \Psi_l \rangle}{M_t M_r P}\right| \geq t\right) \leq 2e^{-L \frac{t^2}{4C_1 + 2C_2 t}} \quad (16)$$

which holds if

$$M_t M_r \geq 2/t \phi_{\theta_n, \theta_l}(M_r) \phi_{\theta_n, \theta_l}(M_t). \quad (17)$$

Case(iv) $\tau_n = \tau_l, \theta_n = \theta_l, f_n \neq f_l$: Consider the absolute value

$$\left|\frac{\langle \Psi_n, \Psi_l \rangle}{M_t M_r P}\right| = \frac{\phi_{f_n, f_l}(P)}{M_t P} \|\mathbf{X}_{\tau_n} \mathbf{v}_t(\theta_n)\|_2^2, \quad (18)$$

where $\phi_{f_n, f_l}(P) \triangleq |\langle \mathbf{D}(f_n), \mathbf{D}(f_l) \rangle|$. It can be viewed as the squared norm of random vector $\tilde{\mathbf{x}} \triangleq \sqrt{\frac{\phi_{f_n, f_l}(P)}{M_t P}} \mathbf{X}_{\tau_n} \mathbf{v}_t(\theta_n)$. The entries in $\tilde{\mathbf{x}}$ are i.i.d zero-mean Gaussian with variance $\sigma_1^2 = \frac{\phi_{f_n, f_l}(P)}{LP}$. Applying the unilateral bound (5a) in Lemma 1 gives

$$\begin{aligned} \Pr\left(\left|\frac{\langle \Psi_n, \Psi_l \rangle}{M_t M_r P}\right| > t\right) &\leq \exp\left(-\frac{1}{L} \left(\frac{t - L\sigma_1^2}{4\sigma_1^2}\right)^2\right) \\ &= \exp\left(-\frac{L}{16} \left(\frac{Pt}{\phi_{f_n, f_l}(P)} - 1\right)^2\right) \leq \exp\left(-\frac{Lt^2}{10}\right) \end{aligned} \quad (19)$$

where the last inequality holds if

$$P \geq \sqrt{2}(1/t + 1) \phi_{f_n, f_l}(P). \quad (20)$$

B. The RIP of The Normalized Ψ

Equipped with the above observations, we are ready to prove the theorem regarding the RIP of the measurement matrix.

Theorem 1: Let $\tilde{\Psi}$ be the normalized measurement matrix, i.e., $\tilde{\Psi} = \Psi / \sqrt{M_t M_r P}$. Then, for any $\delta_K \in (0, 1)$ there exist constant $C_0 \triangleq 3(4C_1 + 2C_2 \delta_K)$, such that $\tilde{\Psi}$ satisfies the RIP of order K with parameter δ_K with probability exceeding $(1 - 4(N_{\tau_n} N_{\theta} N_f)^{-1})$, whenever

$$L \geq C_0 \delta_K^{-2} K^2 \log(N_{\tau} N_{\theta} N_f), \quad (21a)$$

$$M_t M_r \geq 2\delta_K^{-1} K \beta_{\Theta}(M_t, M_r), \quad (21b)$$

$$P \geq \sqrt{2} (\delta_K^{-1} K + 1) \beta_{\mathcal{D}}(P), \quad (21c)$$

where $\beta_{\Theta}(M_t, M_r) \triangleq \sup_{\theta_n, \theta_l \in \Theta, n \neq l} \phi_{\theta_n, \theta_l}(M_t) \phi_{\theta_n, \theta_l}(M_r)$ and $\beta_{\mathcal{D}}(P) \triangleq \sup_{f_n, f_l \in \mathcal{D}, n \neq l} \phi_{f_n, f_l}(P)$.

Proof: The proof of the RIP mainly follows the spirit of the proof in [8]. We only focus on the bounds for the off-diagonal entries in the Gram of $\tilde{\Psi}$. Here we choose

$\delta_d \triangleq \delta_K / K$ and $\delta_o \triangleq (K - 1) \delta_K / K$. The bound on the off-diagonal entries in Case (ii-iv) can be unified using (16) based on the fact that $(4C_1 + 2C_2 t)$ in (16) is always larger than $(4 + 4t)$ in (11) and 10 in (19) for any $t \geq 0$. Substituting t by $\delta_o / (K - 1)$, i.e., δ_K / K , gives

$$\Pr\left(\left|\frac{\langle \Psi_n, \Psi_l \rangle}{M_t M_r P}\right| \geq \frac{\delta_K}{K}\right) \leq 4e^{-\frac{L\delta_K^2}{K^2(4C_1 + 2C_2 \delta_K)}} \quad (22)$$

under conditions in (21b) and (21c), which are derived by substituting $t = \delta_K / K$ into (17) and (20), respectively. The condition in (21a) implies that $\frac{L\delta_K^2}{K^2(4C_1 + 2C_2 \delta_K)} \geq 3 \log(N_{\tau} N_{\theta} N_f)$. Following the steps of the standard scheme [9] proves the RIP. ■

Remark 1: Theorem 1 characterizes the RIP of normalized Ψ under the conditions of (21) for arbitrary array configuration and grid set $\mathcal{T} \times \Theta \times \mathcal{D}$. The condition in (21a) requires that the number of measurements scales quadratically with the number of targets and logarithmically with the number of grid points. The conditions in (21b) and (21c) involve the number of transmitters/receivers, the number of pulses, the number of the targets, the geometry of the grid and the array configuration. This dependence will be explored in the following subsection for minimizing the number of antennas involved.

C. About $\beta_{\mathcal{D}}$ and β_{Θ}

In Theorem 1, the conditions of (21) are very general and can be applied on any array configuration and grid. Next, we look closer at the quantities $\beta_{\mathcal{D}}(P)$ and $\beta_{\Theta}(M_t, M_r)$, which appear in (21).

The quantity $\beta_{\mathcal{D}}(P)$ is determined by the pulse repetition interval, the Doppler grid \mathcal{D} , and the number of pulses. From the definition in (10), it holds that

$$\beta_{\mathcal{D}}(P) \triangleq \sup_{\substack{f_n, f_l \in \mathcal{D} \\ n \neq l}} \left| \frac{\sin(\pi P(f_n - f_l) T_{PRI})}{\sin(\pi(f_n - f_l) T_{PRI})} \right|.$$

Consider the special case where \mathcal{D} is uniform with interval $\Delta_f = \frac{1}{PT_{PRI}}$ and cardinality $|\mathcal{D}| \leq P$. In this case, $\beta_{\mathcal{D}}(P) = 0$, which means that (21c) holds for any K (i.e., K might be larger than P). In order to increase the resolution of \mathcal{D} , we can increase either the number of pulses, or the pulse repetition interval.

The quantity $\beta_{\Theta}(M_t, M_r)$ is determined by the array configuration, the angular grid, Θ , and the number of transmitters/receivers. It is clear that a smaller $\beta_{\Theta}(M_t, M_r)$ is preferable since it requires a smaller $M_t M_r$. Thus, we seek to minimize $\beta_{\Theta}(M_t, M_r)$, or bound it by a small value. For arbitrary transmit/receive array and Θ , it is usually difficult to characterize $\beta_{\Theta}(M_t, M_r)$ analytically. In the CS-MIMO radar literature, some special arrays have been considered. In particular, virtual ULA MIMO radars were considered in [5], where Θ is uniform in the domain $\sin(\theta)$ with $|\Theta| = M_t M_r$ and angular grid interval $\Delta_{\sin(\theta)} = \frac{2}{M_t M_r}$. In this case, $\beta_{\Theta}(M_t, M_r)$ equals 0, which implies that K can be larger than the product $M_t M_r$. However, $M_t M_r$ equals the cardinality of Θ . Random linear array MIMO radars were considered for angle estimation in [7]; in that work, $\frac{\beta_{\Theta}(M_t, M_r)}{M_t M_r}$ is bound

from above by a small value. When the locations of transmit /receive nodes can be assumed to be i.i.d random variables, it was shown in [7, Theorem 2] that $M_t M_r \propto K^2 \log^2(N_\theta)$, which is a special case of Theorem 1 for the case of random array for angle estimation.

In the following, we propose a scheme to minimize the number of receive nodes with respect to the nodes' positions, under the condition of (21b). Given the M_t -element linear transmit array with $y_m^t, \forall m \in \mathbb{N}_{M_t}^+$ and Θ , we would like to solve the following optimization problem:

$$\min_{\mathbf{y}_r} M_r \quad \text{s.t.} \quad \beta_\Theta(M_t, M_r) \leq \frac{M_t M_r \delta_K}{2K} \quad (23)$$

where $\mathbf{y}_r = [y_1^r, \dots, y_{M_r}^r]^T$ denotes the position vector for the receive array. Since the \mathbf{y}_r appears in the exponent of the receive steering vector, the optimization problem in (23) is nonlinear, non-convex. To bypass the difficulty, we formulate the position optimization problem as a relaxed sensor selection problem. Specifically, given a very dense array of receive nodes at positions $\tilde{\mathbf{y}} \triangleq [\tilde{y}_1, \dots, \tilde{y}_M]^T$, we assign a Boolean weight $w_m \in \{0, 1\}$ to each sensor and select the minimum number of sensors by solving the following problem:

$$\begin{aligned} \min_{\mathbf{w}} \quad & \mathbf{1}^T \mathbf{w} \triangleq [1, \dots, 1][w_1, \dots, w_M]^T \\ \text{s.t.} \quad & \sup_{\substack{\theta_n, \theta_l \in \Theta, \\ n \neq l}} \phi_{\theta_n, \theta_l}(M_t) f(\mathbf{w}) \leq \frac{M_t \delta_K}{2K} \mathbf{1}^T \mathbf{w} \\ & \mathbf{1}^T \mathbf{w} \geq 4, w_m \in \{0, 1\}, m \in \mathbb{N}_M^+, \end{aligned} \quad (24)$$

where $f(\mathbf{w}) \triangleq |\langle \mathbf{w}, e^{j2\pi \tilde{\mathbf{y}}(\sin \theta_n - \sin \theta_l)/\lambda} \rangle|$, and constraint $\mathbf{1}^T \mathbf{w} \geq 4$ is imposed to prevent a naive zero vector solution. To obtain a convex relaxation, we replace the non-convex constraints $w_m \in \{0, 1\}$ by the convex constraints $w_m \in [0, 1]$. Then, the relaxed sensor selection problem turns to be

$$\begin{aligned} \min_{\mathbf{w}} \quad & \mathbf{1}^T \mathbf{w} \\ \text{s.t.} \quad & \sup_{\substack{\theta_n, \theta_l \in \Theta, \\ n \neq l}} \phi_{\theta_n, \theta_l}(M_t) f(\mathbf{w}) \leq \frac{M_t \delta_K}{2K} \mathbf{1}^T \mathbf{w} \\ & \mathbf{1}^T \mathbf{w} \geq 4, 0 \leq w_m \leq 1, m \in \mathbb{N}_M^+, \end{aligned} \quad (25)$$

which is a SOCP problem and can be solved efficiently by standard packages. A suboptimal sensor selection set can be generated from \mathbf{w}^* , the optimal solution of (25), by taking its first M_r largest entries.

Remark 2: The proposed optimization scheme in (25) selects the minimum number of receive nodes for CS-MIMO radars satisfying condition (21b) in Theorem 1. It is similar to optimize with respect to the transmit array given fixed receive array. Simulation example in Section IV shows that the CS-MIMO radars generated by the proposed scheme requires much fewer nodes than the ULA array and random array do.

IV. AN EXAMPLE

In this section, we present one example to show the effectiveness of the optimization scheme proposed in Section III-C. We are particularly interested in $K = 3$ targets in the angular

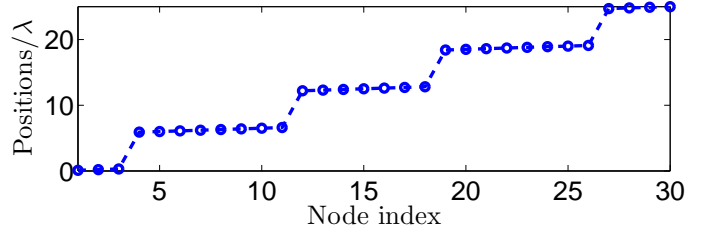


Fig. 1. Positions of the 30 selected nodes by the proposed scheme.

region $\sin \theta \in [0, 0.15]$ discretized uniformly with interval $\Delta_{\sin \theta} = 0.001$. The transmit array is a ULA with $M_t = 40$ nodes and interval 25λ . The receive nodes are chosen from 0.1λ -spaced ULA with 250 nodes. The suboptimal receive array obtained from (25) is nonuniform with $M_r = 30$ nodes. The positions of the nodes are shown in Fig. 1.

For comparison, we know that the virtual array setting in [5] requires a half-wavelength linear receive array with $M_r = 50$ nodes. For random array considered in [7, Theorem 2], $M_t M_r \geq 2121$ is required. It is clear that our method produces CS-MIMO radars with the fewest nodes. We conclude that our proposed optimization scheme relaxes the requirement on $M_t M_r$ in Theorem 1.

V. CONCLUSIONS

We have provided the RIP analysis for CS-MIMO radars with arbitrary array configuration. The conditions involve the number of antennas, targets, transmitted pulses and array geometry. Based on these conditions we have also proposed an antenna selection scheme that minimized the number of receive antennas.

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