MIMO Radar and Communication Spectrum Sharing with Clutter Mitigation

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***Abstract*—We address the co-existence of MIMO radars and a MIMO communication system. Unlike previous works, we consider a scenario in which the radar system operates in the presence of clutter. Both the radar and the communication system use transmit precoding. Initially, spectrum sharing is formulated as a problem that maximizes the radar SINR subject to the communication system meeting certain rate and power constraints. Due to the dependence of the clutter on the radar precoding matrix, the optimization with respect to the radar precoder is a maximization of a nonconvex function over a nonconvex feasible set. Since solving such problem is computationally intractable, we propose to maximize a lower bound of the SINR. In the resulting alternating maximization problem, the alternating iteration of the communication TX covariance matrix reduces to one SDP problem, while the iteration of the radar precoder is solved by a sequence of SOCP problems, which are more efficient and tractable than SDP. Simulation results validate the effectiveness of the proposed spectrum sharing method for scenarios with clutter.**

***Index Terms*—MIMO radar, spectrum sharing, clutter mitigation, SOCP**

I. Introduction

The operating frequency bands of communication and radar systems often overlap, causing one system to exert interference to the other. For example, the high UHF radar systems overlap with GSM communication systems, and the S-band radar systems partially overlap with Long Term Evolution (LTE) and WiMax systems [1]–[4]. Spectrum sharing targets at enabling radar and communication systems to share the spectrum efficiently by minimizing interference effects [3]–[11]. Spectrum sharing between MIMO radar and communication systems has been considered in [4]–[7], where the radar interference to the communication system is eliminated by projecting the radar waveforms onto the null space of the interference channel from radar to communication systems. Spatial filtering at the radar receiver is proposed in [8] to reject interference from the communication systems. The existing radar-communication spectrum sharing literature addresses interference mitigation either for the communication systems [4]–[7], or for the radar [8]. To the best of our knowledge, co-design of radar and communication systems for spectrum sharing was proposed in [12]–[15] for the first time. Compared to radar design approaches of [4]–[8], the joint design has the potential to improve the spectrum utilization due to increased number of design degrees of freedom. However, a clutter free scenario was assumed in both works of [12]–[15] and [4]–[8].

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In this paper, we consider the co-design based spectrum sharing of a MIMO radar and a communication system for a scenario in which the radar system operates in the presence of clutter. Both the radar and the communication system use transmit precoding. Initially, spectrum sharing is formulated as a problem that maximizes the radar SINR subject to the communication system meeting certain rate and power constraints. Usually, the joint design problem can be solved using alternating optimization. Due to the dependence of the clutter on radar precoding matrix, the optimization with respect to (w.r.t.) the radar precoder is a maximization of a nonconvex function over a nonconvex feasible set. Solving such problem is computationally intractable and demanding. In addition, the objective is also nonlinear and nonconvex w.r.t. the communication covariance matrix. The joint design problem requires to solve a sequence of semidefinte programming (SDP) problems in every alternating iteration of either design variable; as such it has high computational complexity. As an efficient alternative, we propose to maximize a lower bound of the SINR. In the resulting alternating maximization problem, the alternating iteration of the communication covariance matrix reduces to one SDP problem. We show that the radar precoder always has a rank one solution. Based on this key observation, the alternating iteration of the radar precoder is solved by a sequence of second order cone programming (SOCP) problems, which are more efficient and tractable than SDP problems. Simulation results validate the effectiveness of the proposed spectrum sharing method for scenarios with clutter.

The paper is organized as follows. Section II introduces the coexistence model of a MIMO radar system and a communication system. The proposed spectrum sharing method is given in Section III. Numerical results and conclusions are provided respectively in Sections IV and V. *Notation:* $\mathcal{CN}(\mu, \mathbf{\Sigma})$ denotes the circularly symmetric complex Gaussian distribution with mean μ and covariance matrix $\mathbf{\Sigma}$. $|\cdot|$ and $\mathrm{Tr}(\cdot)$ denote the matrix determinant and trace respectively. The set $\mathbb{N}_L^+$ is defined as $\{1, \ldots, L\}$. δ_{ij} denotes the Kronecker delta. $\lfloor x \rfloor$ denotes the largest integer not larger than x. $\mathbf{A}^T$ and $\mathbf{A}^H$ respectively denote the transpose and Hermitian transpose.

II. System Models

Consider a MIMO communication system which coexists with a MIMO radar system as shown in Fig. 1, sharing the same carrier frequency. The MIMO radar system uses

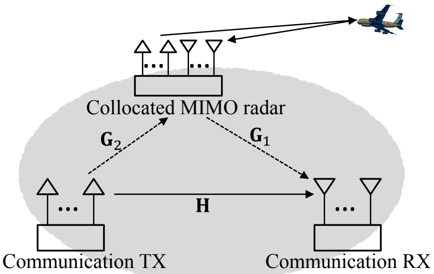

Fig. 1. A MIMO communication system sharing spectrum with a colocated MIMO radar system

$M_{t,R}$ TX and $M_{r,R}$ RX collocated antennas for target detection/estimation. The communication transmitter and receiver are equipped with $M_{t,C}$ and $M_{r,C}$ antennas, respectively. The communication channel is denoted as $\mathbf{H} \in \mathbb{C}^{M_{r,C} \times M_{t,C}}$. The interference channel from the radar TX antennas to the communication receiver is denoted as $\mathbf{G}_1 \in \mathbb{C}^{M_{r,C} \times M_{t,R}}$ [4], [5], [7]; the interference channel from the communication transmitter to the radar RX antennas is denoted as $\mathbf{G}_2 \in \mathbb{C}^{M_{r,R} \times M_{t,C}}$. It is assumed that the channels $\mathbf{H}$, $\mathbf{G}_1$ and $\mathbf{G}_2$ are block fading [16] and perfectly known at the communication transmitter. In practice, the channel state information can be obtained through the transmission of pilot signals [4], [17]. The detailed signal models for the MIMO radar and communication systems are described in the sequel. We do not assume perfect carrier phase synchronization between the two systems.

The MIMO radar employs narrowband orthogonal waveforms, each of which contains L coded sub-pulses, each of duration T_b. Let $\mathbf{s}_m \triangleq [s_{m1}, \ldots, s_{mL}]^T$ denote the orthogonal code vector for the m-th TX antenna. It holds that $\langle \mathbf{s}_m, \mathbf{s}_n \rangle = \delta_{mn}$. In this paper, we choose $\mathbf{S}$ as a random orthonormal matrix [14], which is obtained through performing the Gram-Schmidt orthogonalization on a matrix whose entries are i.i.d. Gaussian random variables. Note that the entries of $\mathbf{S}$ are not independent anymore. However, based on [18, Theorem 3], if $M_{t,R} = \mathcal{O}(L/\ln L)$, the entries of $\mathbf{S}$ can be approximated by i.i.d. Gaussian random variables with distribution $\mathcal{N}(0, 1/L)$. The waveforms are first precoded by matrix $\mathbf{P} \in \mathbb{C}^{M_{t,R} \times M_{t,R}}$, and then transmitted over carrier f_c periodically, with pulse repetition interval T_{PRI}. Suppose that there are one target and K point clutters in the same range bin w.r.t. the radar phase center. During each pulse, the target echoes and communication interference received at the radar RX antennas are demodulated to baseband and sampled every T_b seconds. The discrete time signal model for sampling time index $l \in \mathbb{N}_{\tilde{L}}^+$ is expressed as

$$\begin{aligned} \mathbf{y}_R(l) &= \beta_0 \mathbf{v}_r(\theta_0) \mathbf{v}_t^T(\theta_0) \mathbf{P}\mathbf{s}(l - l_0) + \mathbf{G}_2 \mathbf{x}(l) e^{j\alpha_2(l)} \\ &+ \sum_{k=1}^{K} \beta_k \mathbf{v}_r(\theta_k) \mathbf{v}_t^T(\theta_k) \mathbf{P}\mathbf{s}(l - l_0) + \mathbf{w}_R(l), \end{aligned} \quad (1)$$

where $\tilde{L} = \lfloor T_{PRI}/T_b \rfloor$ denotes the total number of samples in one PRI; $\mathbf{y}_R(l)$ and $\mathbf{x}(l)$ respectively denote the radar received signal and communication waveform symbol at time lT_b; $\mathbf{s}(l) = [s_{1l}, \ldots, s_{M_{t,R}l}]^T$; $\mathbf{w}_R(l)$ is noise distributed as $\mathcal{CN}(\mathbf{0}, \sigma_R^2 \mathbf{I})$; $l_0 = \lfloor \tau_0/T_b \rfloor$ with $\tau_0 \triangleq 2d_0/v_c$, d_0 being the range of the target and v_c being the speed of light; β_0 and β_k, $\forall k \in \mathbb{N}_K^+$, denote the complex radar cross sections for the target and the k-th point clutter, respectively; the Swerling II target model is assumed, *i.e.*, the β_0 varies from pulse to pulse and has distribution $\mathcal{CN}(0, \sigma_{\beta 0}^2)$; and $\mathbf{v}_r(\theta) \in \mathbb{C}^{M_{r,R}}$ is the receive steering vector defined as

$$\mathbf{v}_r(\theta) \triangleq \left[e^{j2\pi \langle \mathbf{d}_1^r, \mathbf{u}(\theta) \rangle / \lambda_c}, \ldots, e^{j2\pi \langle \mathbf{d}_{M_{r,R}}^r, \mathbf{u}(\theta) \rangle / \lambda_c} \right]^T,$$

with $\mathbf{d}_m^r \triangleq [x_m^r \; y_m^r]^T$ denoting the two-dimensional coordinates of the m-th RX antenna, $\mathbf{u}(\theta) \triangleq [\cos(\theta) \; \sin(\theta)]^T$, and λ_c denoting the carrier wavelength. $\mathbf{v}_t(\theta) \in \mathbb{C}^{M_{t,R}}$ is the transmit steering vector and is respectively defined. The second term on the right hand side of (1) denotes the interference due to the communication transmission $\mathbf{x}(l) \in \mathbb{C}^{M_{t,C}}$. $e^{j\alpha_2(l)}$ is introduced to denote the random phase offset resulted from the random phase jitters of the oscillators at the communication transmitter and the MIMO radar receiver Phase-Locked Loops [13]. In the literature [19]–[21], phase jitters are modeled as zero-mean Gaussian processes.

The MIMO communication system uses the same carrier frequency f_c. The baseband signal at the communication receiver is sampled according to the symbol rate T_s, which could be different that the radar waveform symbol duration T_b. In this paper, we only consider the matched case, *i.e.*, $T_s = T_b$; the extension of the proposed methods to the mismatched case is straightforward [13]. The discrete time communication signal has the following form

$$\mathbf{y}_C(l) = \mathbf{H}\mathbf{x}(l) + \mathbf{G}_1 \mathbf{P}\mathbf{s}(l) e^{j\alpha_1(l)} + \mathbf{w}_C(l), \; l \in \mathbb{N}_{\tilde{L}}^+, \quad (2)$$

where $\mathbf{x}(l) \in \mathbb{C}^{M_{t,C}}$ denotes the transmit vector at the communication transmitter at time index l; $e^{j\alpha_1(l)}$ denotes the random phase offset between the radar TX carrier and the communication RX reference carrier [13]; the additive noise $\mathbf{w}_C(l)$ has distribution $\mathcal{CN}(\mathbf{0}, \sigma_C^2 \mathbf{I})$. Note that the radar waveform $\mathbf{s}(l)$ equals zero when $l > L$, which means that the communication system is interference free during this period. The above model assumes that the radar transmission is the only interference, while the target and clutter returns do not reach the communication system.

III. Spectrum Sharing with Clutter Mitigation

We first derive the communication rate and radar SINR in terms of communication and radar waveforms.

For the communication system, the covariance of interference plus noise is given by

$$\mathbf{R}_{\mathrm{C}inl} = \begin{cases} \mathbf{G}_1 \mathbf{\Phi} \mathbf{G}_1^H + \sigma_C^2 \mathbf{I} & l \in \mathbb{N}_L^+ \\ \sigma_C^2 \mathbf{I} & l \in \mathbb{N}_{\tilde{L}}^+ \setminus \mathbb{N}_L^+ \end{cases} \quad (3)$$

where $\mathbf{\Phi} \triangleq \mathbf{P}\mathbf{P}^H/L$ is positive semidefinite. For $l \in \mathbb{N}_L^+$, the *instaneous* information rate is unknown because the interference plus noise is not necessarily Gaussian due to the random phase offset $\alpha_1(l)$. Instead, we are interested in a lower bound of the rate, which is given by [22]

$$\underline{C}(\mathbf{R}_x, \mathbf{\Phi}) \triangleq \log_2 \left| \mathbf{I} + \mathbf{R}_{\mathrm{C}inl}^{-1} \mathbf{H} \mathbf{R}_x \mathbf{H}^H \right|,$$

which is achieved when the codeword $\mathbf{x}(l)$, $l \in \mathbb{N}_L^+$ is distributed as $\mathcal{CN}(0, \mathbf{R}_x)$. The average communication rate over $\tilde{L}$ symbols is as follows

$$\mathrm{C}_{\mathrm{avg}}(\mathbf{R}_x, \mathbf{\Phi}) \triangleq \eta \underline{C}(\mathbf{R}_x, \mathbf{\Phi}) + (1 - \eta) \underline{C}(\mathbf{R}_x, \mathbf{0}), \quad (4)$$

where $\eta \triangleq L/\tilde{L}$ is called the radar duty cycle.

For the radar system, the covariance of the communication interference exerted at the radar RX antennas equals $\mathbb{E}\{\mathbf{G}_2\mathbf{x}(l)e^{j\alpha_2(l)}e^{-j\alpha_2(l)}\mathbf{x}^H(l)\mathbf{G}_2^H\} = \mathbf{G}_2\mathbf{R}_x\mathbf{G}_2^H$. Suppose that each of the clutter amplitude β_k is an independent complex Gaussian variable with zero mean and variance $\sigma_{\beta k}^2$. The above clutter model is widely considered in the literature [23]–[25]. The clutter covariance matrix is given as $\mathbf{R}_c = \sum_{k=1}^{K}\mathbf{C}_k\mathbf{\Phi}\mathbf{C}_k^H$ with $\mathbf{C}_k = \sigma_{\beta k}\mathbf{v}_r(\theta_k)\mathbf{v}_t^T(\theta_k)$. Incorporating the additive noise and interference from both clutter and the communication system, the radar SINR is given as

$$\text{SINR}(\mathbf{R}_x,\mathbf{\Phi}) = \text{Tr}\left((\mathbf{R}_{\text{Rin}}+\mathbf{R}_c)^{-1}\mathbf{D}_0\mathbf{\Phi}\mathbf{D}_0^H\right), \tag{5}$$

where $\mathbf{R}_{\text{Rin}} \triangleq \mathbf{G}_2\mathbf{R}_x\mathbf{G}_2^H + \sigma_R^2\mathbf{I}$ and $\mathbf{D}_0 = \sigma_{\beta 0}\mathbf{v}_r(\theta_0)\mathbf{v}_t^T(\theta_0)$.

Here we consider the scenario where the radar searches in particular directions of interest given by set $\{\theta_k\}$ for targets with unknown RCS variances and delays [23], [26]. The worst possible target RCS variance is given by $\{\sigma_0^2\}$, which is the smallest target RCS variance that could be detected by the radar. We assume that $\{\sigma_{\beta k}^2\}$ and $\{\theta_k\}$ are known. In practice, these clutter parameters could be estimated when target is absent [24].

The spectrum sharing problem when clutter is present can be formulated as follows

$$(\mathbf{P}_1) \max_{\mathbf{R}_x\succeq 0,\mathbf{\Phi}\succeq 0} \text{SINR}, \text{ s.t. } \text{C}_{\text{avg}}(\mathbf{R}_x,\mathbf{\Phi}) \geq C, \tag{6a}$$

$$\tilde{L}\text{Tr}(\mathbf{R}_x) \leq P_C, L\text{Tr}(\mathbf{\Phi}) \leq P_R. \tag{6b}$$

Note that the objective in $(\mathbf{P}_1)$ is not affine w.r.t. $\mathbf{\Phi}$ because the clutter covariance in SINR depends on $\mathbf{\Phi}$. One natural solution is the sequential convex programming using the first order Taylor expansion of the SINR. Solving the sequence of approximated problems increases the computational complexity. It is even worse when the sequential convex programming is embedded in every alternating iterations w.r.t. $\mathbf{R}_x$ and $\mathbf{\Psi}$.

In this paper, we propose a more efficient alternative where we maximize a lower bound of the SINR. To tackle the nonconvexity in the objective function, we propose to maximize a lower bound of the objective function

$$\text{SINR} \geq \frac{\sigma_{\beta 0}^2 M_{r,R}^2 \text{Tr}(\mathbf{\Phi}\mathbf{D}_t)}{\text{Tr}(\mathbf{\Phi}\mathbf{C})+\text{Tr}(\mathbf{R}_x\mathbf{B})+\sigma_R^2 M_{r,R}}, \tag{7}$$

where $\mathbf{D}_t \triangleq \mathbf{v}_t^*(\theta_0)\mathbf{v}_t^T(\theta_0)$, $\mathbf{C} \triangleq \sum_{k=1}^{K}\mathbf{C}_k^H\mathbf{v}_r(\theta_0)\mathbf{v}_r^H(\theta_0)\mathbf{C}_k$ and $\mathbf{B} \triangleq \mathbf{G}_2^H\mathbf{v}_r(\theta_0)\mathbf{v}_r^H(\theta_0)\mathbf{G}_2$. The lower bound is derived based on Cauchy-Schwarz inequality and is tight if the clutter plus interference is spectrally white, *i.e.*, $(\mathbf{R}_{\text{Rin}}+\mathbf{R}_c) \propto \mathbf{I}$. The approximate problem is now given as

$$(\mathbf{P}_1') \max_{\mathbf{R}_x\succeq 0,\mathbf{\Phi}\succeq 0} \frac{\sigma_{\beta 0}^2 M_{r,R}^2 \text{Tr}(\mathbf{\Phi}\mathbf{D}_t)}{\text{Tr}(\mathbf{\Phi}\mathbf{C})+\text{Tr}(\mathbf{R}_x\mathbf{B})+\sigma_R^2 M_{r,R}}, \tag{8}$$
$$\text{s.t. same constraints as}(\mathbf{P}_1).$$

Alternate optimization is applied to solve $(\mathbf{P}_1')$. The alternating iterations w.r.t. $\mathbf{R}_x$ and $\mathbf{\Phi}$ are discussed in the following two subsections.

A. The Alternating Iteration w.r.t. $\mathbf{R}_x$

With fixed $\mathbf{\Phi}$, the optimization w.r.t. $\mathbf{R}_x$ can be formulated as the following equivalent convex problem

$$\min_{\mathbf{R}_x\succeq 0} \text{Tr}(\mathbf{R}_x\mathbf{B}) \text{ s.t. } \text{C}_{\text{avg}}(\mathbf{R}_x,\mathbf{\Phi}) \geq C, \tilde{L}\text{Tr}(\mathbf{R}_x) \leq P_C. \tag{9}$$

Problem (9) can be solved using available SDP solvers [27].

B. The Alternating Iteration w.r.t. $\mathbf{\Phi}$

For the optimization of $\mathbf{\Phi}$ with fixed $\mathbf{R}_x$, the constraint in (6a) is nonconvex w.r.t. $\mathbf{\Phi}$. The first order Taylor expansion of $\text{C}_{\text{avg}}(\mathbf{R}_x,\mathbf{\Phi})$ at $\bar{\mathbf{\Phi}}$ is given as

$$\text{C}_{\text{avg}}(\mathbf{R}_x,\mathbf{\Phi}) \approx \text{C}_{\text{avg}}(\mathbf{R}_x,\bar{\mathbf{\Phi}}) - \text{Tr}\left[\mathbf{A}(\mathbf{\Phi}-\bar{\mathbf{\Phi}})\right],$$

where $\mathbf{A}$ is given in (10) on the top of next page.

The sequential convex programming technique is applied to solve $\mathbf{\Phi}$ by repeatedly solve the following approximate optimization problem

$$\max_{\mathbf{\Phi}\succeq 0} \frac{\text{Tr}(\mathbf{\Phi}\mathbf{D}_t)}{\text{Tr}(\mathbf{\Phi}\mathbf{C})+\rho}, \text{ s.t. } \text{Tr}(\mathbf{\Phi}\mathbf{A}) \leq \tilde{C}/L, \text{Tr}(\mathbf{\Phi}) \leq P_R/L. \tag{11}$$

where $\tilde{C} = L[\underline{C}(\mathbf{R}_x,\bar{\mathbf{\Phi}})+\text{Tr}(\bar{\mathbf{\Phi}}\mathbf{A})]+(\tilde{L}-L)\underline{C}(\mathbf{R}_x,\mathbf{0})-\tilde{L}C$, $\rho = \text{Tr}(\mathbf{R}_x\mathbf{B})+\sigma_R^2 M_{r,R}$ are real positive constants w.r.t. $\mathbf{\Phi}$, and $\bar{\mathbf{\Phi}}$ is updated as the solution of the previous repeated problem. Problem (11) could be formulated as a semidefinite programming problem (SDP) via Charnes-Cooper Transformation [24], [28]. However, in each alternating iteration w.r.t. $\mathbf{\Phi}$, it is required to solve several iterations of SDP due to the sequential convex programming, which could be computational demanding if $\mathbf{\Phi}$ has large dimensions. In the following, we will show that (11) always has rank one solution, and thus it could be further solved using more efficient second order conic programming (SOCP). To do so, we introduce the following SDP problem

$$\min_{\mathbf{\Phi}\succeq 0} \text{Tr}(\mathbf{\Phi}) \text{ s.t. } \text{Tr}(\mathbf{\Phi}\mathbf{A}) \leq \tilde{C}/L, \frac{\text{Tr}(\mathbf{\Phi}\mathbf{D}_t)}{\text{Tr}(\mathbf{\Phi}\mathbf{C})+\rho} \geq \gamma, \tag{12}$$

where γ is a real positive constant. The following proposition relates the optimal solutions of problems (11) and (12).

Proposition 1. *If γ in (12) is chosen to be the maximum achievable SINR of (11), denoted as SINR_{max}, the optimal $\mathbf{\Phi}$ of (12) is also optimal for (11).*

Proof: Denote $\mathbf{\Phi}_1^*$ and $\mathbf{\Phi}_2^*$ the optimal solutions of (11) and (12), respectively. It is clear that $\mathbf{\Phi}_1^*$ is feasible point of (12). This means that $\text{Tr}(\mathbf{\Phi}_2^*) \leq \text{Tr}(\mathbf{\Phi}_1^*) \leq P_R/L$. Therefore, $\mathbf{\Phi}_2^*$ is a feasible point of (11). It holds that

$$\text{SINR}_{\text{max}} \equiv \frac{\text{Tr}(\mathbf{\Phi}_1^*\mathbf{D}_t)}{\text{Tr}(\mathbf{\Phi}_1^*\mathbf{C})+\rho} \geq \frac{\text{Tr}(\mathbf{\Phi}_2^*\mathbf{D}_t)}{\text{Tr}(\mathbf{\Phi}_2^*\mathbf{C})+\rho} \geq \text{SINR}_{\text{max}}.$$

It is only possible when all the equalities hold. In other words, $\mathbf{\Phi}_2^*$ is optimal for (11). The claim is proved. ∎

Based on the above proposition, the optimal solution of (11) can be obtained by solving (12) using a bisection search for γ. Given an interval $[l,u]$ which contains SINR_{max}, we start from solving (12) with $\gamma = \frac{l+u}{2}$. If the optimal solution of (12) is feasible for (11), this means that SINR_{max} is larger than $\frac{l+u}{2}$, and the interval is updated by its upper half; otherwise, the interval is updated by its lower half. The above procedure is repeated until the interval is sufficient small. The remaining issue is to find an algorithm that solves (12) more efficiently than SDP does.

In order to characterize the optimal solution of (12), we need the following key lemma:

Lemma 1. *Matrix $\mathbf{A}$ defined in (10) is positive semidefinite.*

$$\mathbf{A} \triangleq -\left(\frac{\partial C_{\text{avg}}(\mathbf{R}_x, \mathbf{\Phi})}{\partial \Re(\mathbf{\Phi})}\right)^T_{\mathbf{\Phi}=\bar{\mathbf{\Phi}}} = \mathbf{G}_1^H[(\mathbf{G}_1\mathbf{\Phi}\mathbf{G}_1^H + \sigma_C^2\mathbf{I})^{-1} - (\mathbf{G}_1\mathbf{\Phi}\mathbf{G}_1^H + \sigma_C^2\mathbf{I} + \mathbf{H}\mathbf{R}_x\mathbf{H}^H)^{-1}]\mathbf{G}_1\big|_{\mathbf{\Phi}=\bar{\mathbf{\Phi}}}. \tag{10}$$

Proof: For simplicity of notation, we denote that $\mathbf{X} \triangleq \mathbf{G}_1\mathbf{\Phi}\mathbf{G}_1^H + \sigma_C^2\mathbf{I} \succ 0$ and $\mathbf{Y} \triangleq \mathbf{H}\mathbf{R}_x\mathbf{H}^H \succeq 0$. It is easy to see that $\mathbf{A}$ is Hermitian because both $\mathbf{X}^{-1}$ and $(\mathbf{X}+\mathbf{Y})^{-1}$ are Hermitian. It is sufficient to show that $\mathbf{M} \triangleq \mathbf{X}^{-1} - (\mathbf{X}+\mathbf{Y})^{-1}$ is positive semidefinite. We have that

$$\mathbf{X}^{-1} - (\mathbf{X}+\mathbf{Y})^{-1} = \mathbf{X}^{-1}\mathbf{Y}(\mathbf{X}+\mathbf{Y})^{-1},$$

which could be shown by right multiplying $(\mathbf{X}+\mathbf{Y})$ on both sides of the equality. Since $\mathbf{X}$, $\mathbf{Y}$ and $\mathbf{M}$ are Hermitian, we have

$$\mathbf{M} = \mathbf{X}^{-1}\mathbf{Y}(\mathbf{X}+\mathbf{Y})^{-1} = (\mathbf{X}+\mathbf{Y})^{-1}\mathbf{Y}\mathbf{X}^{-1}.$$

Since $(\mathbf{X}+\mathbf{Y})^{-1}$ is invertible, there exists a unique positive definite matrix $\mathbf{V}$, such that $(\mathbf{X}+\mathbf{Y})^{-1} = \mathbf{V}^2$. Simple algebra manipulation shows that

$$\begin{aligned}\mathbf{V}^{-1}\mathbf{M}\mathbf{V}^{-1} &= (\mathbf{V}^{-1}\mathbf{X}^{-1}\mathbf{V}^{-1})(\mathbf{V}\mathbf{Y}\mathbf{V}) \\ &= (\mathbf{V}\mathbf{Y}\mathbf{V})(\mathbf{V}^{-1}\mathbf{X}^{-1}\mathbf{V}^{-1}),\end{aligned}$$

i.e., $\mathbf{V}^{-1}\mathbf{M}\mathbf{V}^{-1}$ is a product of two commutable positive semidefinite matrices $\mathbf{V}^{-1}\mathbf{X}^{-1}\mathbf{V}^{-1}$ and $\mathbf{V}\mathbf{Y}\mathbf{V}$. Therefore, $\mathbf{V}^{-1}\mathbf{M}\mathbf{V}^{-1}$ and thus $\mathbf{M}$ is positive semidefinite. ∎

Based on Lemma 1, we prove the following result by following the approach in [28]:

Proposition 2. *Suppose that (12) is feasible. Then, the optimal solution of (12) must be rank one and unique. Moreover, (11) always has rank one solution.*

Proof: Problem (12) is an SDP, whose Karush-Kuhn-Tucker (KKT) conditions are given as

$$\mathbf{\Psi} + \lambda_2\mathbf{D}_t = \mathbf{I} + \lambda_1\mathbf{A} + \lambda_2\gamma\mathbf{C} \tag{13a}$$

$$\mathbf{\Psi}\mathbf{\Phi} = \mathbf{0} \tag{13b}$$

$$\mathbf{\Psi} \succeq 0, \mathbf{\Phi} \succeq 0, \lambda_1 \geq 0, \lambda_2 \geq 0 \tag{13c}$$

$$\text{Tr}(\mathbf{\Phi}\mathbf{D}_t) \geq \gamma\text{Tr}(\mathbf{\Phi}\mathbf{C}) + \gamma\rho \tag{13d}$$

where $\mathbf{\Psi} \succeq 0, \lambda_1 \geq 0, \lambda_2 \geq 0$ are dual variables. From (13a), we have

$$\text{rank}(\mathbf{\Psi}) + \text{rank}(\lambda_2\mathbf{D}_t) \geq \text{rank}(\mathbf{I} + \lambda_1\mathbf{A} + \lambda_2\gamma\mathbf{C}).$$

Recall that $\text{rank}(\mathbf{D}_t) = 1$. Since $\mathbf{A}$ and $\mathbf{C}$ are PSD, the matrix on right hand side of (13a) has full rank. Therefore, $\text{rank}(\mathbf{\Psi})$ is not smaller than $M_{t,R}-1$. From (13b) and (13d) we conclude that the optimal $\mathbf{\Phi}$ must be a rank one matrix.

The uniqueness of optimal solution could be proved via contradiction. The second claim on the solution of (11) follows from Proposition 1. ∎

Proposition 2 implies that when there is only one target, the transmit beamforming is the optimal radar precoding strategy for the spectrum sharing between the MIMO radar and the communication systems along with clutter mitigation for radar, as formulated in $(\mathbf{P}_1')$. Based on Proposition 2, we denote that $\mathbf{\Phi} = \mathbf{u}\mathbf{u}^H$, where $\mathbf{u}$ is a vector of dimension $M_{t,R}$. Problem (12) can be reformulated as

$$\begin{aligned}&\min_{\mathbf{u}} \|\mathbf{u}\|^2 \text{ s.t. } \|\mathbf{A}^{1/2}\mathbf{u}\|^2 \leq \tilde{C}/L, \\ &\qquad \gamma\mathbf{u}^H\mathbf{C}\mathbf{u} + \gamma\rho \leq \left(\mathbf{u}^H\mathbf{v}_t^*(\theta_0)\right)^2.\end{aligned} \tag{14}$$

Note that if $\mathbf{u}$ is a solution of (14), so is $e^{jw}\mathbf{u}$ for any real w. Without loss of generality, we restrict $\mathbf{u}^H\mathbf{v}_t^*(\theta_0)$ is real and nonnegative. Problem (14) is equivalent to the following SOCP

$$\begin{aligned}&\min_{\mathbf{u},t} t \text{ s.t. } \|\mathbf{u}\|_2 \leq t, \left\|\mathbf{A}^{1/2}\mathbf{u}\right\|_2 \leq \sqrt{\tilde{C}/L}, \\ &\left\|\begin{bmatrix}\gamma\mathbf{C} & \\ & \gamma\rho\end{bmatrix}^{1/2}\begin{bmatrix}\mathbf{u} \\ 1\end{bmatrix}\right\|_2 \leq \mathbf{u}^H\mathbf{v}_t^*(\theta_0).\end{aligned} \tag{15}$$

The proposed efficient spectrum sharing algorithm in presence of clutter using a lower bound of the radar SINR is outlined in Algorithm 1.

Algorithm 1 The proposed algorithm for spectrum sharing with clutter mitigation $(\mathbf{P}_1')$.

1: **Input:** $\mathbf{D}_0, \mathbf{C}_n, \mathbf{H}, \mathbf{G}_1, \mathbf{G}_2, P_{C/R}, C, \sigma_{C/R}^2, \delta_1$
2: **Initialization:** $\mathbf{\Phi} = \frac{P_R}{LM_{t,R}}\mathbf{I}$, $\mathbf{R}_x = \frac{P_C}{\tilde{L}M_{t,C}}\mathbf{I}$;
3: **repeat**
4: Update $\mathbf{R}_x$ by solving (9) with fixed $\mathbf{\Phi}$;
5: Update $\mathbf{\Phi}$ by solving a sequence of approximated problem (11), which is in turn achieved by bisection search and repeatedly solving (15) using SOCP solvers;
6: **until** $|\text{SINR}^n - \text{SINR}^{n-1}| < \delta_1$
7: **Output:** $\mathbf{R}_x, \mathbf{P} = \mathbf{u}$

IV. Simulation Results

In this section, we provide two simulation examples to quantify the performance of the proposed spectrum sharing method with clutter mitigation. We set the number of samples per PRI to $\tilde{L} = 32$, the number of radar waveform symbols to $L = 8$, the noise variance to $\sigma_C^2 = \sigma_R^2 = 0.01$, and the number of antennas to $M_{t,R} = M_{r,R} = 16, M_{t,C} = 8, M_{r,C} = 4$. The MIMO radar system consists of collocated TX and RX antennas forming half-wavelength uniform linear arrays. The radar waveforms are chosen from the rows of a random orthonormal matrix [12]. There are one stationary targets with RCS variance $\sigma_{\beta 0}^2 = 5 \times 10^{-5}$ and eight point clutters. All clutter RCS variances are set to be identical and are denoted by σ_β^2, which is decided by the prescribed clutter to noise ratio (CNR) $10\log\sigma_\beta^2/\sigma_R^2$. The target angle θ_0 w.r.t. the array is randomly generated; clutter scatters are with angles in $[\theta_0 - 20°, \theta_0 - 10°]$ and $[\theta_0 + 10°, \theta_0 + 20°]$. For the communication capacity and power constraints, we take $C = 24$ bits/symbol and $P_C = \tilde{L}M_{t,C}$ (the power is normalized by the power of the radar waveform). The interference channels $\mathbf{G}_1$ and $\mathbf{G}_2$ are generated with entries which are independent and distributed as $\mathcal{CN}(0, 0.1)$. The channel $\mathbf{H}$ has independent entries, distributed as $\mathcal{CN}(0,1)$. The proposed spectrum sharing method with clutter mitigation jointly designs the communication covariance matrix and the radar precoder according to the algorithm presented in Section III. For comparison, we also implement the method based on the Charnes-Cooper transformation of (11) and SDP. The aforementioned spectrum sharing algorithms are respectively labeled by "precoding with clutter mitigation (SOCP)" and "precoding with clutter mitigation (SDP)" in the figures. We also implement the spectrum sharing method without the consideration of clutter mitigation, labeled by "precoding without clutter mitigation", and method based on uniform precoding, *i.e.*, $\mathbf{P} = \sqrt{LP_R/M_{t,R}}\mathbf{I}$.

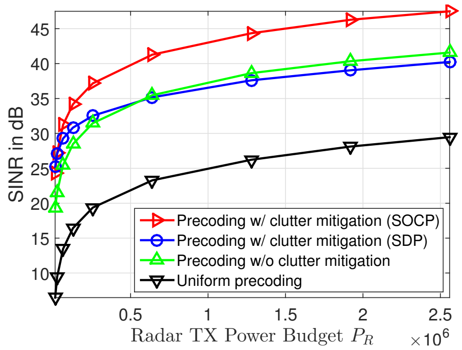

Fig. 2. SINR performance under different values of radar TX power.

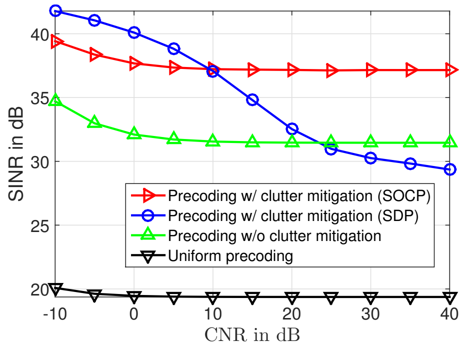

Fig. 3. SINR performance under different clutter to noise ratios (CNR).

Fig. 2 shows the SINR results for different values of the radar transmit power budget P_R. The CNR is fixed as 20 dB. The radar power budget per antenna ranges from 100 to $20,000$ times of the communication power budget per antenna. Fig. 3 shows the SINR results under different clutter to noise ratios. The radar power budget is fixed as $P_R = 2.56 \times 10^5$. We can observe that the proposed method achieves the highest SINR while the uniform precoding based method achieves the lowest SINR. The method "precoding without clutter mitigation" improves the SINR over uniform precoding because it focuses more power on the target. Our proposed method achieves higher SINR than the method without clutter mitigation because our method can effectively reduce the power transmitted on the clutter. Note that the performance of the SDP based method degrades greatly as the CNR increases, even worse than the spectrum sharing method without considering clutter mitigation. This indicates that the SDP based method is very sensitive to CNR. A rigorous treatment on this phenomenon will be considered in the future work.

Comparing with the spectrum sharing method using SDP based precoding design, our proposed SOCP based precoding design is more tractable and computationally efficient. From Fig. 2 and Fig. 3, we can see that the proposed method outperforms the SDP based method when CNR is larger than 10 dB. The CPU time required by the SDP method increase dramatically with $M_{t,R}$, while the proposed SOCP based method increase mildly with $M_{t,R}$.

V. Conclusion

We have proposed an efficient spectrum sharing method for a MIMO radar and a communication system operating in a scenario with clutter. The radar and communication system signals were optimally designed by minimizing a lower bound for the SINR at the radar receive antennas. We have shown that the radar precoder always has a rank one solution. Based on this key observation, the alternating iteration of the radar precoder has been solved by a sequence of SOCP problems, which are more efficient and tractable than applying SDP directly. Simulation results have shown that the proposed spectrum sharing method can effectively increase the radar SINR for various scenarios with clutter.

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