Performance Guarantees for Distributed MIMO Radar based on Sparse Sensing

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***Abstract*—Sparse sensing-based distributed MIMO radars exploit the sparsity of the targets in the discretized target space to achieve the good target estimation performance of MIMO radars but with fewer measurements. Based on sparse sensing, the problem of target estimation is formulated as a sparse signal recovery problem, where the signal to be recovered is block sparse, or equivalently, the sensing matrix is block-diagonal and the signal to be recovered consists of equal size blocks that have the same sparsity profile. This paper develops the theoretical requirements and performance guarantees for the application of sparse recovery techniques to this problem. The obtained theoretical results confirm that exploiting the block sparsity of the target in the target space can reduce the number of measurements needed for target estimation, or can result in improved target estimation for the same number of samples.**

***Index Terms*—Distributed MIMO radar, sparse sensing, restricted isometry property, block diagonal matrices.**

I. Introduction

Multiple-input multiple-output (MIMO) radars [1], [2], [3] have received considerable attention in recent years due to their improved performance over traditional phase arrays. Distributed MIMO radars are a special class of MIMO radars in which the antennas are widely separated. Due to the spatial diversity, introduced by the multiple independents paths between the targets and the transmit/receive antennas, distributed MIMO radars enjoy improved target estimation performance as compared to phased arrays. By exploiting the sparsity of targets in the target space, sparse sensing [4], [5] has been introduced in distributed MIMO radars [6], [7], allowing them to maintain their good target estimation performance while involving fewer data. Based on sparse sensing, the problem of target estimation is formulated as a sparse signal recovery problem, where the signal to be recovered is block sparse. The block sparsity arises in [6], [7] by grouping together in the measurement matrix transmit/receive antenna pair measurements corresponding to the same grid point. Exploiting block sparsity results in improved target estimation and reduction of the number of measurements needed. However, in [6], [7] the performance of the recovery algorithms was addressed via simulations only.

This paper considers the aforementioned problem, and by permuting the columns of the measurement matrix we reformulate the block-sparse signal recovery problem into a problem in which the measurement matrix $\mathbf{\Psi}$ is block diagonal (BD) and the sparse signal, s, contains equal-sized blocks that have the same sparsity profile. This reformulation enables us to perform restricted isometry property (RIP)-based performance analysis. Target estimation, or equivalently, the recovery of s can be achieved by the mixed ℓ_2/ℓ_1-optimization program (L-OPT) of [8]. The effectiveness of L-OPT is guaranteed if the RIP of $\mathbf{\Psi}$ holds with respect to sparse signals with the aforementioned structure. We show that if the number of measurements at each receiver scales quadratically with the number of the targets and logarithmically with the number of grid points in the location-speed space, then $\mathbf{\Psi}$ satisfies the required RIP.

The derived theoretical results confirm that the BD structure in $\mathbf{\Psi}$ and the sparsity structure in s reduce the number of measurements needed for target estimation in sparse modeling based distributed MIMO radars. In addition, the RIP-based analysis in this paper provides a uniform recovery guarantee, which means that once $\mathbf{\Psi}$ satisfies the RIP, target estimation can be achieved with high probability even in the worst case.

Relations to prior work: Existing theoretical work on sparse modeling based MIMO radar considered collocated antennas [9], [10]. These works, however, do not easily extend to the distributed MIMO radar scenario. While the RIP analysis technique for measurement matrix in this paper is related to that for a Toeplitz matrix, presented in [11], our analysis deals with a block-diagonal measurement matrix with additional complex exponential factors introduced by the moving targets.

The paper is organized as follows. In Section II, we derive the sparse model of a MIMO radar system with widely separated antennas. In Section III, the performance of L-OPT recovery algorithm is provided assuming that the measurement matrix satisfies the $\mathcal{A}$-RIP, which is established in Section IV. Simulation results are given in Section V.

II. Signal Model

We consider a MIMO radar system with M_t transmit nodes (TX) and M_r receive nodes (RX), which are widely separated. Let (x_i^t, y_i^t) and (x_i^r, y_i^r) denote the locations of the i-th transmit and receive antenna in cartesian coordinates, respectively. The i-th TX antenna transmits repeated pulses with pulse repetition interval T. Each pulse contains the continuous-time waveform $x_i(t)e^{j2\pi f_i t}$, where f_i is the carrier frequency. Let us assume that there are K moving targets present in the space. For simplicity, we consider a clutter-free environment.

This work was supported by ONR under Grant N00014-12-1-0036.

The location-speed space is discretized by $\mathbf{\Theta} \equiv \{(x_n, y_n, v_x^n, v_y^n), n = 1, \ldots, N\}$, $N \triangleq N_x \times N_y \times N_{v_x} \times N_{v_y}$, and it is assumed that the targets fall on grid points.

Suppose that the j-th receive antenna obtains L T_s-spaced samples from each pulse transmitted by antenna i. On stacking the samples from P pulses into vector $\mathbf{z}_{ij}$ it holds [7]:

$$\mathbf{z}_{ij} = \mathbf{\Psi}_{ij}\mathbf{s}_{ij} + \mathbf{n}_{ij} \tag{1}$$

where $\mathbf{s}_{ij} = \left[s_{ij}^1, \ldots, s_{ij}^N\right]^T$, with s_{ij}^n being non-zero only if there is a target at the n-th grid point (here n refers to a particular ordering of grid points of the 4-dimensional space into a vector of length N); $\mathbf{n}_{ij}$ represents noise; and

$$\mathbf{\Psi}_{ij} = \begin{bmatrix} \mathbf{x}_{i,\tau_{ij}^1} e^{j2\pi f_{ij}^1 T} & \cdots & \mathbf{x}_{i,\tau_{ij}^N} e^{j2\pi f_{ij}^N T} \\ \vdots & \ddots & \vdots \\ \mathbf{x}_{i,\tau_{ij}^1} e^{j2\pi f_{ij}^1 PT} & \cdots & \mathbf{x}_{i,\tau_{ij}^N} e^{j2\pi f_{ij}^N PT} \end{bmatrix}_{(LP)\times N} \tag{2}$$

where $\mathbf{x}_{i,\tau_{ij}^n}$ is a vector that contains L samples of the i-th waveform appropriately shifted by τ_{ij}^n. τ_{ij}^n and f_{ij}^n respectively denote the propagation time and Doppler frequency associated with the n-th grid and the TX/RX antenna pair (i, j).

$$f_{ij}^n = \frac{\left\langle (v_x^n, v_y^n), \mathbf{d}_{in}^t \right\rangle}{\lambda_i \|\mathbf{d}_{in}^t\|_2} + \frac{\left\langle (v_x^n, v_y^n), \mathbf{d}_{in}^r \right\rangle}{\lambda_i \|\mathbf{d}_{in}^r\|_2} \tag{3}$$

where $\mathbf{d}_{in}^{t/r} \triangleq ((x_i^{t/r}, y_i^{t/r}) - (x_n, y_n))$ denotes the vector from the n-th grid to the i-th TX/RX antenna, and λ_i is the carrier wavelength of the i-th transmitter.

On stacking the received samples from all TX/TR antenna pairs into a column vector $\mathbf{z}$ of length LPM_tM_r, we get

$$\mathbf{z} = \left[(\mathbf{z}_{11})^T, \ldots, (\mathbf{z}_{M_tM_r})^T\right]^T = \mathbf{\Psi}\mathbf{s} + \mathbf{n} \tag{4}$$

where $\mathbf{s} = \left[(\mathbf{s}_{11})^T, \ldots, (\mathbf{s}_{M_tM_r})^T\right]^T$, $\mathbf{n} = [(\mathbf{n}_{11})^T, \ldots, (\mathbf{n}_{M_tM_r})^T]^T$, and $\mathbf{\Psi} = \text{diag}(\mathbf{\Psi}_{11}, \ldots, \mathbf{\Psi}_{M_tM_r})$.

Note that each vector $\mathbf{s}_{ij}$ contains zero entries except the entries corresponding to grid points occupied by targets. Thus, the vector $\mathbf{s}$ is a concatenation of M_tM_r subvectors that share the same sparsity profile, and have exactly K nonzero entries each. We can say that $\mathbf{s}$ lies in $\mathcal{A}_0^K$ defined by

$$\begin{aligned} \mathcal{A}_0^K \triangleq \{\mathbf{y} \in & \mathbb{R}^{NM_tM_r} : \text{supp}(\mathbf{y}_1) = \cdots = \text{supp}(\mathbf{y}_{M_tM_r}), \\ & |\text{supp}(\mathbf{y}_j)| \leq K, \, j = 1, \ldots, M_tM_r\} \end{aligned} \tag{5}$$

where $\mathbf{y}_j \in \mathbb{R}^N$, $j = 1, \ldots, M_tM_r$ are uniformly partitioned blocks of $\mathbf{y}$, supp$(\cdot)$ and $|\cdot|$ denote the index set of nonzero entries of a vector, and the cardinality of a set, respectively. In the next section, the target estimation, or equivalently the recovery of vector $\mathbf{s}$, is achieved by adapting a block sparse recovery method in compressive sensing literature.

III. The Sparse Recovery Algorithm

To apply the mixed ℓ_2/ℓ_1-optimization program (L-OPT) of [8] for the sparse recovery, we can permute the columns of $\mathbf{\Psi}$ and correspondingly the entries of $\mathbf{s}$ to generate block sparsity in the target vector. Specifically, $\mathbf{s}$ is recovered by solving

$$\min \sum_{n=1}^{N} \|\mathbf{s}[\mathcal{I}_n]\|_2 \;\; s.t. \; \|\mathbf{z} - \mathcal{P}_M(\mathbf{\Psi})\mathcal{P}_v(\mathbf{s})\|_2 \leq \epsilon. \tag{6}$$

where $\mathcal{P}_M$ is the column permutation matrix applied on $\mathbf{\Phi}$ and $\mathcal{P}_v$ the corresponding permutation operator applied on $\mathbf{s}$; $\{\mathcal{I}_n\}_{n=1}^N$ are the sets with cardinality M_tM_r containing the indices of the n-th entries from all blocks $\mathbf{s}_{ij}$; ϵ is related to the norm of vector $\mathbf{n}$. In the above, $\mathcal{P}_v(\mathbf{s}) = [\mathbf{s}[\mathcal{I}_1, \ldots, \mathcal{I}_N]]^T$ is block-sparse. In the following we will denote by $\mathcal{A}_{blk}^K$ the set of block-sparse vectors resulting from permutations of $\mathbf{s} \in \mathcal{A}_0^K$, i.e., $\mathcal{A}_{blk}^K \triangleq \{\mathcal{P}_v(\mathbf{s}) \,|\, \mathbf{s} \in \mathcal{A}_0^K\}$.

In [8], the authors provided the recovery performance guarantee by (6) given the condition that $\mathcal{P}_M(\mathbf{\Psi})$ satisfies the $\mathcal{A}_{blk}$-RIP which is defined as follows.

Definition 1 ([12]): For set $\mathcal{A}$, $\mathbf{\Psi}$ is said to satisfy the $\mathcal{A}$-restricted isometry property with constant $\delta \in (0, 1)$, in short, $\mathcal{A}$-RIP(K, δ), if δ is the smallest value such that $(1-\delta)\|\mathbf{s}\|_2^2 \leq \|\mathbf{\Psi}\mathbf{s}\|_2^2 \leq (1+\delta)\|\mathbf{s}\|_2^2$ holds for all $\mathbf{s} \in \mathcal{A}$.

Note that by setting $t = 1$ in Lemma 2 in the Appendix, the norm of vector $\mathbf{n}$ is upper bounded by $\epsilon \triangleq 2\sqrt{LPM_tM_r\sigma_n^2}$ with probability at least $(1 - p_1)$ where $p_1 = e^{-c_0 LPM_tM_r}$. Substituting the expression of ϵ into the result of [8, Theorem 2] gives the following performance guarantees.

Performance of sparse sensing-based MIMO radars: Suppose that the transmitted waveforms are bandlimited Gaussian signals with variance σ_0^2. $\tilde{\mathbf{\Psi}} \triangleq \mathbf{\Psi}/\sqrt{LP\sigma_0^2}$ is with unit-norm columns. If $\mathcal{P}_M(\tilde{\mathbf{\Psi}})$ satisfies the $\mathcal{A}_{blk}$-RIP$(2K, \delta_{2K})$ with $\delta_{2K} \leq \sqrt{2}-1$, then the L-OPT method in (6) can recover $\mathbf{s}$ with

$$\|\hat{\mathbf{s}} - \mathbf{s}\|_{\ell_2} \leq \frac{8\sqrt{M_tM_r}\sqrt{1+\delta_{2K}}}{1-(1+\sqrt{2})\delta_{2K}} \frac{\sigma_n}{\sigma_0} \tag{7}$$

and with probability at least $(1 - p_1)$.

The above result assumes that $\mathcal{P}_M(\tilde{\mathbf{\Psi}})$ satisfies $\mathcal{A}_{blk}$-RIP. However, $\mathcal{P}_M(\tilde{\mathbf{\Psi}})$ has a complicated structure, which makes the RIP analysis difficult. In [13, Proposition 1], we have shown that the $\mathcal{A}_{blk}$-RIP of $\mathcal{P}_M(\tilde{\mathbf{\Psi}})$ is equivalent to the $\mathcal{A}_0$-RIP of $\tilde{\mathbf{\Psi}}$. However, still, establishing the $\mathcal{A}_0$-RIP of $\tilde{\mathbf{\Psi}}$ directly is difficult. However, we can follow an indirect way to establish the $\mathcal{A}_0$-RIP of $\tilde{\mathbf{\Psi}}$. Let us define

$$\begin{aligned} \mathcal{A}_1^K \triangleq \{\mathbf{y} \in & \mathbb{R}^{NM_tM_r} : |\text{supp}(\mathbf{y}_1)| = \cdots = |\text{supp}(\mathbf{y}_{M_tM_r})|, \\ & |\text{supp}(\mathbf{y}_j)| \leq K, \, j = 1, \ldots, M_tM_r\}. \end{aligned} \tag{8}$$

One can see that $\mathcal{A}_0^K \subset \mathcal{A}_1^K$. If we can show that $\tilde{\mathbf{\Psi}}$ satisfies the $\mathcal{A}_1$-RIP$(2K, \delta_{2K})$, then $\tilde{\mathbf{\Psi}}$ will satisfy $\mathcal{A}_0$-RIP$(2K, \delta_{2K}^0)$ with δ_{2K}^0 smaller than δ_{2K}. That is to say that the L-OPT achieves the performance in (7) if $\tilde{\mathbf{\Psi}}$ satisfies the $\mathcal{A}_1$-RIP. We claim that $\tilde{\mathbf{\Psi}}$ satisfies the $\mathcal{A}_1$-RIP under certain conditions as shown in the next section.

IV. The $\mathcal{A}_1$-RIP of The Measurement Matrix

Ahead of the RIP analysis, we provide some observations on the Gram of matrix $\tilde{\mathbf{\Psi}}$. Let us first state one lemma which will be used later.

Lemma 1: Let $\{x_i\}$ and $\{y_i\}$, $i = 1, \ldots, Q$ be sequences of identical distributed zero-mean Gaussian variables with variance σ^2. All variables are independent except that the last I ($I \in [1, Q)$) variables of $\{x_i\}$ are the first I variables of $\{y_i\}$, i.e., $x_{i+Q-I} = y_i$ for any $i \in [1, I]$. Then

$$\Pr\left(\left|\sum_{i=1}^{Q} x_i y_i\right| \geq t\right) \leq 4\exp\left(-\frac{(Q-1)t^2}{8Q\sigma^2(Q\sigma^2 + t/2)}\right).$$

We know that $\{x_i y_i\}_{i=1}^{Q}$ are not mutually independent. Lemma 1 can be proved by a splitting trick, similar to [11].

A. *Observations on The Gram of $\tilde{\mathbf{\Psi}}$*

The Gram of $\tilde{\mathbf{\Psi}}$, denoted here by $\mathbf{G}$, is also block-diagonal, i.e., $\mathbf{G} = \text{diag}(\mathbf{G}_{11}, \ldots, \mathbf{G}_{M_t M_r})$ where $\mathbf{G}_{ij} = \tilde{\mathbf{\Psi}}_{ij}^H \tilde{\mathbf{\Psi}}_{ij}$ and $\tilde{\mathbf{\Psi}}_{ij} \triangleq \mathbf{\Psi}_{ij}/\sqrt{LP\sigma_0^2}$.

Consider the (n, m)th entry in $\mathbf{G}_{ij}$. It holds that

$$\begin{aligned} G_{ij}(n,m) &= \frac{\mathbf{x}_{i,\tau_{ij}^n}^T \mathbf{x}_{i,\tau_{ij}^m}}{LP\sigma_0^2} \sum_{p=1}^{P} e^{j2\pi(f_{ij}^m - f_{ij}^n)(pT)} \\ &= \frac{\mathbf{x}_{i,\tau_{ij}^n}^T \mathbf{x}_{i,\tau_{ij}^m}}{LP\sigma_0^2} \frac{\sin\big(\pi(f_{ij}^m - f_{ij}^n)TP\big)}{\sin\big(\pi(f_{ij}^m - f_{ij}^n)T\big)} e^{j2\pi(f_{ij}^m - f_{ij}^n)PT} \end{aligned} \tag{9}$$

which is the inner product of the columns in $\tilde{\mathbf{\Psi}}_{ij}$ corresponding to the nth and mth grid points. The following three cases are analyzed:

Case (i) For $n = m$, i.e., the diagonal entries, $G_{ij}(n) = \frac{1}{L\sigma_0^2}\mathbf{x}_{i,\tau_{ij}^n}^T \mathbf{x}_{i,\tau_{ij}^n}$ which is the sum of squares of i.i.d Gaussian variables with $\mathbb{E}\{G_{ij}(n)\} = 1$. It holds that (see also [11])

$$\Pr(|G_{ij}(n) - 1| > t) \leq 2\exp\big(-\frac{Lt^2}{16}\big). \tag{10}$$

Case(ii) the nth and mth grid points have different coordinates ($\tau_{ij}^n \neq \tau_{ij}^m$). From (9), it holds that $\mathbb{E}\{G_{ij}(n,m)\} = 0$ and

$$|G_{ij}(n,m)| \leq \frac{\rho_{ij}}{L\sigma_0^2}|\mathbf{x}_{i,\tau_{ij}^n}^T \mathbf{x}_{i,\tau_{ij}^m}| \tag{11}$$

where ρ_{ij} is the maximum value that $\left|\frac{1}{P}\frac{\sin\left(\pi(f_{ij}^m - f_{ij}^n)TP\right)}{\sin\left(\pi(f_{ij}^m - f_{ij}^n)T\right)}\right|$ can achieve ($\rho_{ij} \in [0,1]$). We consider the quantity $|G_{ij}(n,m)| = |G_{ij}(n,m) - \mathbb{E}\{G_{ij}(n,m)\}|$. In order to give a bound on $|G_{ij}(n,m)|$, it suffices to consider the worst case, because

$$\begin{aligned} \Pr\big(|G_{ij}(n,m)| > t\big) &\leq \Pr\left(\frac{\rho_{ij}}{L\sigma_0^2}|\mathbf{x}_{i,\tau_{ij}^n}^T \mathbf{x}_{i,\tau_{ij}^m}| > t\right) \\ &\leq \Pr\left(\frac{1}{L\sigma_0^2}|\mathbf{x}_{i,\tau_{ij}^n}^T \mathbf{x}_{i,\tau_{ij}^m}| > t\right). \end{aligned} \tag{12}$$

Now, we need to provide the bound on the inner product of $\mathbf{x}_{i,\tau_{ij}^n}$ and $\mathbf{x}_{i,\tau_{ij}^m}$. Note that $\mathbf{x}_{i,\tau_{ij}^n}$ and $\mathbf{x}_{i,\tau_{ij}^m}$ are both truncated out from the i-th waveform and may share some common entries. The general bound of [11, Lemma 5] referring to two distinct i.i.d random vectors cannot be applied directly. Applying Lemma 1 for each off-diagonal entry $G_{ij}(n,m)$, we have

$$\Pr\big(|G_{ij}(n,m)| > t\big) \leq 4\exp\left(-\frac{(L-1)t^2}{8(1+t/2)}\right). \tag{13}$$

Case(iii) the nth and mth grid points share the same coordinates ($\tau_{ij}^n = \tau_{ij}^m$) but have different speeds ($f_{ij}^n \neq f_{ij}^m$). Consider the absolute value

$$|G_{ij}(n,m)| = \frac{\mathbf{x}_{i,\tau_{ij}^n}^T \mathbf{x}_{i,\tau_{ij}^n}}{LP\sigma_0^2}\left|\frac{\sin\big(\pi(f_{ij}^m - f_{ij}^n)TP\big)}{\sin\big(\pi(f_{ij}^m - f_{ij}^n)T\big)}\right|. \tag{14}$$

If we denote the second multiplier as C_{ij}^{mn}, $|G_{ij}(n,m)|$ can be viewed as the squared norm of $\sqrt{\frac{C_{ij}^{mn}}{LP\sigma^2}}\mathbf{x}_{i,\tau_{ij}^n}$ with i.i.d zero-mean Gaussian entries with variance $\sigma_1^2 = \frac{C_{ij}^{mn}}{LP}$. Applying the unilateral bound in Lemma 5 in [11], we have

$$\begin{aligned} \Pr\left(|G_{ij}(n,m)| > t\right) &\leq \exp\left(-\frac{1}{L}\left(\frac{t - L\sigma_1^2}{4\sigma_1^2}\right)^2\right) \\ &= \exp\left(-\frac{L}{16}\left(\frac{Pt}{C_{ij}^{nm}} - 1\right)^2\right) \leq \exp\left(-\frac{L}{16}\left(\frac{t}{\gamma_{ij}} - 1\right)^2\right) \end{aligned} \tag{15}$$

where

$$\begin{aligned} \gamma_{ij} &= \sup_{(m,n)\in\mathcal{S}_2} \left|C_{ij}^{mn}/P\right|, \\ \mathcal{S}_2 &\triangleq \{(m,n) | m,n \leq N, \tau_{ij}^n = \tau_{ij}^m, f_{ij}^n \neq f_{ij}^m\}. \end{aligned} \tag{16}$$

B. *$\mathcal{A}_1$-RIP of the Normalized Measurement Matrix*

Equipped with the above observations, we are ready to prove the following theorem regarding the $\mathcal{A}_1$-RIP of $\tilde{\mathbf{\Psi}}$.

Theorem 1: Let $\tilde{\mathbf{\Psi}}$ be the normalized measurement matrix with $M_t M_r \geq 3$, i.e., $\tilde{\mathbf{\Psi}} = \mathbf{\Psi}/\sqrt{LP\sigma_0^2}$. Then, for any $\delta_K \in (0,1)$ there exist constants c_1 and c_2 depending only on δ_K, such that whenever

$$L \geq c_2 K^2 \log(N M_t M_r) + 1, \tag{17}$$

and $\gamma_{ij} \leq \delta_K/(2K + \delta_K)$ for all $i \in [1, M_t]$, $j \in [1, M_r]$ defined in (16), $\tilde{\mathbf{\Psi}}$ satisfies $\mathcal{A}_1$-RIP(K, δ_K) with probability exceeding $1 - \exp(-c_1(L-1)/K^2)$. Specifically, for any $c_1 \leq \delta_K^2/64$, it suffices to choose $c_2 \geq 128/(\delta_K^2 - 64c_1)$.

Proof: Here we only focuss on the bounds for the off-diagonal entries in the Gram of $\tilde{\mathbf{\Psi}}$, $\mathbf{G} = \tilde{\mathbf{\Psi}}^H \tilde{\mathbf{\Psi}}$. For diagonal entries, i.e., $n = m$ as in case (i), the union bound can be easily obtained based on (10).

The off-diagonal entries may be from either case (ii) or case (iii). In order to arrive at a uniform union bound, we need to unify the bounds in (13) and (15) for these two cases. Inequality (13) for case (ii) can be relaxed as

$$\Pr\left(|G_{ij}(n,m)| > t\right) \leq 4\exp\left(-\frac{L-1}{16}t^2\right). \tag{18}$$

With $1/\gamma_{ij} = 1/t + \Delta$, (15) for case (iii) becomes

$$\begin{aligned} \Pr\left(|G_{ij}(n,m)| > t\right) &\leq \exp\left(-\frac{L}{16}(1 + \Delta t - 1)^2\right) \\ &\leq \exp\left(-\frac{L}{16}\Delta^2 t^2\right) \leq 4\exp\left(-\frac{L-1}{16}\Delta^2 t^2\right). \end{aligned} \tag{19}$$

If $\Delta^2 \geq 1$, i.e., $(1/\gamma_{ij} - 1/t)^2 \geq 1$, the above bound for case (iii) turns to be the same as that in (18) for case (ii). Considering the fact that small γ_{ij} is required to guarantee that the off-diagonal elements in case (iii) are small, we have the constraint $\gamma_{ij} \leq t/(1+t)$.

Note that there are only K instead of KM_tM_r off-diagonal entries contributing to the radius of the Gergosin's disc. This reduction comes from the BD structure of $\tilde{\mathbf{\Psi}}$ and the sparsity profile of $\mathbf{s}$ characterized by $\mathcal{A}_1^K$. Therefore, substituting t by δ_o/K, we can easily arrive at the union bound for all off-diagonal entries under constraint $\gamma_{ij} \leq \delta_o/(K+\delta_o)$. Following the steps of the standard scheme [11] proves the theorem. ■

Remarks: Note that we exploit the sparsity structures in $\tilde{\mathbf{\Psi}}$ and $\mathbf{s}$ when applying Gergošin's Disc Theorem for off-diagonal entries. To emphasize the advantage of the block-sparse structure in our scenario, we compare to a scenario in which the block-structure is ignored, and the recovery is based on a full Toeplitz matrix of size $LM_tM_r \times NM_tM_r$ and a sparse vector with KM_tM_r nonzero entries at arbitrary locations. From [11], a full Toeplitz matrix satisfies the RIP if L is on the order of $\mathcal{O}(K^2M_tM_r\log(NM_tM_r))$, which is M_tM_r times larger than the bound in (17). Comparing that to (17) suggests that exploiting the block sparsity reduces the number of samples needed. This validates previous simulation-based observations in [7], suggesting that exploiting the structure in both $\mathbf{\Psi}$ and $\mathbf{s}$ allows for reduction of the number of samples, L, needed for target estimation.

V. Numerical Results

We consider a MIMO radar system with $M_t = 2$ TX and $M_r = 2$ RX antennas, distributed uniformly on a circle of radius of $6,000m$ and $3,000m$, respectively. Each TX radar transmits pulses with interval 0.125 ms and $5GHz$ carrier frequency. Each RX radar works with sampling frequency of 5MHz. The signal-to-noise ratio SNR is defined as $10\log_{10}(\sigma_0^2/\sigma_n^2)$. The probing space is discretized on a 20×4 grid, with grid spacing equal to 10 m. The velocity space is $V_x \in [100, 130]m/s$, $V_y = 100m/s$ and is uniformly discretized on a 4×1 grid. The dimension of the target vector is 1280. The grid is kept small so that the complexity is manageable. We randomly generate K targets on the grid. The reflection coefficient for each target is set to 1.

We first illustrate the choice of the number of pulses P via the inequality $\gamma_{ij} \leq \delta_{2K}/(4K+\delta_{2K})|_{\delta_{2K}\leq\sqrt{2}-1} \triangleq \gamma_0$. Fig. 1(a) shows values of γ_{ij} for all TX/RX pairs under different values of P. We choose the smallest P that guarantees that all γ_{ij}'s are smaller than γ_0, i.e., $P = 12$. Based on Theorem 1, this value guarantees the performance under the worst cases. In the following simulation, we will show that even a smaller P works well too.

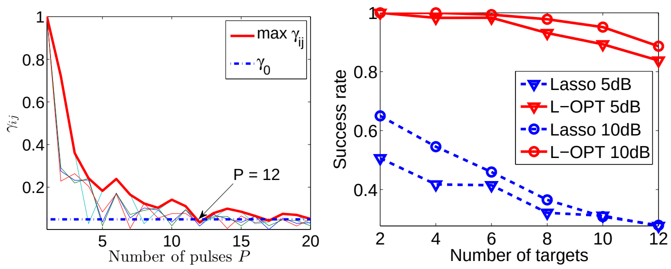
In Fig. 1(b), we plot the successful recovery rate for different number of targets K with $L = 6, P = 3$. For comparison, we also implement Lasso which does not exploit the sparsity structure in $\mathbf{s}$. The success rate decreases as K increases. As implied by the theorems, L-OPT outperforms Lasso because it exploits the sparsity structure in $\mathbf{s}$.

VI. Conclusions

We have considered moving target estimation using distributed, sparsity based MIMO radars. We have provided the uniform recovery guarantee by analyzing the $\mathcal{A}$-RIP of the block diagonal measurement matrix. The proposed theoretical results validate that the structures in both $\mathbf{\Psi}$ and $\mathbf{s}$ result in reduction of the number of measurements needed, or result in improved target estimation for the same L.

VII. Appendix

Lemma 2 ((1.14) in [14]): Let $\mathbf{x} \triangleq [x_1, \ldots, x_Q]^T$ be an Gaussian random vector with i.i.d entries that have distribution $\mathcal{N}(0, \sigma^2)$. There exists a constant $c_0 > 0$ such that for any $t > 0$ it holds that $\Pr\left(\|\mathbf{x}\|_2 \geq (1+t)\sqrt{Q}\sigma\right) \leq \exp(-c_0Qt^2)$.



(a) $\max_{ij}\gamma_{ij}$ v.s. P (b) Success rate v.s. K

Fig. 1. Results on the choice of the number of pulses, P, and success recovery rate for different number of targets, K.

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