Proof of Proposition 1 in "A Joint Design Approach for Spectrum Sharing between Radar and Communication Systems"

We state Proposition 1 in [1] as follows:

Proposition 1 ([1]). *Suppose that* $\{\mathbf{R}_{xl}\}$ *is initialized by* $\{\mathbf{R}_{xl}\} \equiv \mathbf{R}_x^0$. *Then, the optimal value of* ($\mathbf{P}_R$) *in every iteration of the proposed algorithm could be achieved by* $\{\mathbf{R}_{xl}^n\}$ *such that for any* $l, l' \in \mathbb{N}_L^+$ *(or* $l, l' \in \mathbb{N}_{\tilde{L}}^+ \setminus \mathbb{N}_L^+$*), it holds that* $\mathbf{R}_{xl}^n = \mathbf{R}_{xl'}^n$.

Proof: The proposition can be proved using induction. We focus on the proof for $l, l' \in \mathbb{N}_L^+$ in the following. The proof for $l, l' \in \mathbb{N}_{\tilde{L}}^+ \setminus \mathbb{N}_L^+$ is similar.

From the proposition, we know that $\{\mathbf{R}_{xl}\}$ is initialized such that $\mathbf{R}_{xl}^0 = \mathbf{R}_{xl'}^0 = \mathbf{R}_x^0$, $\forall l, l' \in \mathbb{N}_L^+$. We need to show that the optimal value of ($\mathbf{P}_R$) in the n-th iteration is also achieved by $\{\mathbf{R}_{xl}^n\}$ such that $\mathbf{R}_{xl}^n = \mathbf{R}_{xl'}^n$, $\forall l, l' \in \mathbb{N}_L^+$. Because $\{\mathbf{R}_{xl}^n\}$ is obtained via several inner iterations of solving ($\tilde{\mathbf{P}}_{\mathbf{R}}$) in [1], it suffices to show that the above property could be passed on between the iterations of solving ($\tilde{\mathbf{P}}_{\mathbf{R}}$).

Suppose that, in the $(i-1)$-th inner iteration, the optimal value of ($\tilde{\mathbf{P}}_R$) is achieved by $\{\mathbf{R}_{xl}^{n(i-1)}\}$ such that $\mathbf{R}_{xl}^{n(i-1)} = \mathbf{R}_{xl'}^{n(i-1)}$, $\forall l, l' \in \mathbb{N}_L^+$. During the i-th iteration, $\{\mathbf{R}_{xl}^{ni*}\}$ is obtained by solving ($\tilde{\mathbf{P}}_R$) with $\{\bar{\mathbf{R}}_{xl}\} = \{\mathbf{R}_{xl}^{n(i-1)}\}$. We will show that $\mathbf{R}_{xl}^{ni} \equiv 1/L \sum_{l=1}^{L} \mathbf{R}_{xl}^{ni*} \triangleq \mathbf{R}_x^{ni}$, $\forall l \in \mathbb{N}_L^+$ is also feasible and achieves the same radar SINR as $\{\mathbf{R}_{xl}^{ni*}\}_{l \in \mathbb{N}_L^+}$ does. Based on the concavity of $C_l(\mathbf{R}_{xl}, \cdot)$, we have

$$\sum_{l=1}^{L} C_l(\mathbf{R}_{xl}^{ni*}, \cdot) \leq L C_l\left(\mathbf{R}_x^{ni}, \cdot\right).$$

For the communication transmission power, it trivially holds that $\sum_{l=1}^{L} \mathrm{Tr}(\mathbf{R}_{xl}^{ni*}) = L\mathrm{Tr}(\mathbf{R}_x^{ni})$. Therefore, $\{\mathbf{R}_x^{ni}\}$ is also feasible. The objective $\tilde{f}(\mathbf{R}_{xl})$ of ($\tilde{\mathbf{P}}_R$) in the i-th iteration is affine w.r.t. $\mathbf{R}_{xl}$ with coefficient depending on $\bar{\mathbf{R}}_{xl} = \mathbf{R}_{xl}^{n(i-1)}$. Given that $\mathbf{R}_{xl}^{n(i-1)} = \mathbf{R}_{xl'}^{n(i-1)}$, $\forall l, l' \in \mathbb{N}_L^+$, the affine functions $\tilde{f}(\cdot)$ for are identical for any $l \in \mathbb{N}_L^+$. Therefore, $\{\mathbf{R}_x^{ni}\}$ achieves the same objective value as $\{\mathbf{R}_{xl}^{ni*}\}_{l \in \mathbb{N}_L^+}$ does.

Proposition 1 is proved. ■

References

[1] B. Li, H. Kumar, and A. P. Petropulu, "A joint design approach for spectrum sharing between radar and communication systems," in *submitted to IEEE International Conference on Acoustics, Speech and Signal Processing*, March 2016.