# Spectrum Sharing Between Matrix Completion Based MIMO Radars and A MIMO Communication System

Bo Li and Athina Petropulu

April 23, 2015

ECE Department, Rutgers, The State University of New Jersey, USA

Work supported by NSF under Grant ECCS-1408437

# Outline

- Motivation
- Existing spectrum sharing approaches
- Introduction to the Matrix Completion based MIMO (MIMO-MC) Radar
- The Coexistence Signal Model
- Spectrum Sharing based on Optimum Communication Waveform Design
  - Interference to the MIMO-MC Radar
  - Two Spectrum Sharing Approaches
  - Comparison
- Spectrum Sharing based on Joint Communication and Radar System Design
- Simulation Results

# Motivation

- Spectrum is a limited resource. Spectrum sharing can increase the spectrum efficiency.
- Radar and communication system overlap in the spectrum domain thus causing interference to each other.

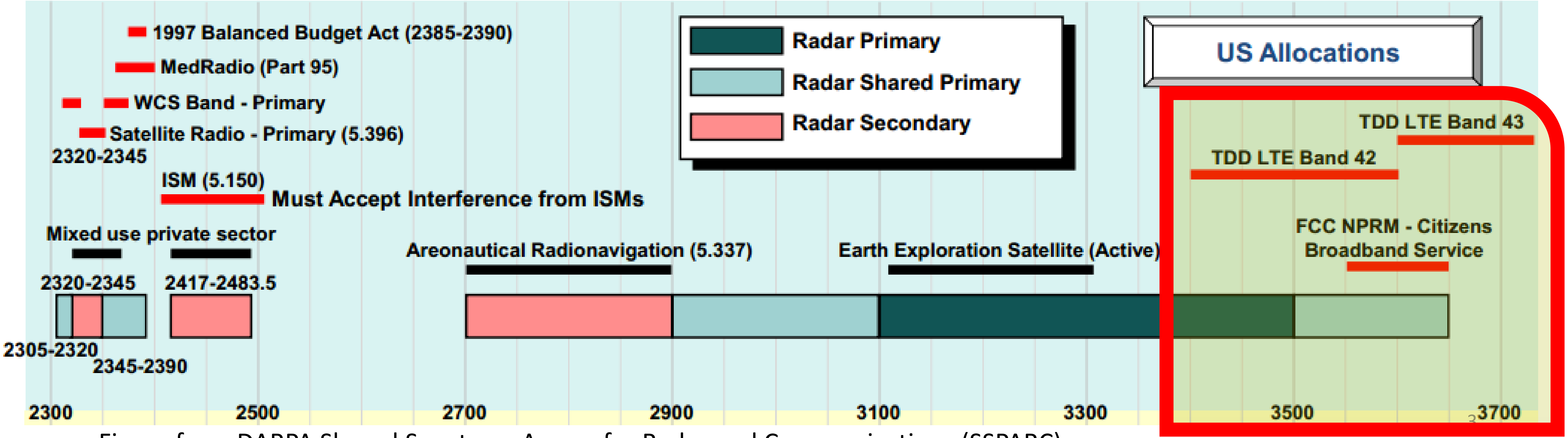



Figure from DARPA Shared Spectrum Access for Radar and Communications (SSPARC)

# Motivation

- Matrix completion based MIMO radar (MIMO-MC) [Sun and Petropulu, 14] is a good candidate for reducing interference at the radar receiver.
  - Traditional MIMO radars transmit orthogonal waveforms from their multiple transmit (TX) antennas, and their receive (RX) antennas forward their measurements to a fusion center to populate a “data matrix” for further processing.
  - Based on the low-rankness of the data matrix, MIMO-MC radar RX antennas forward to the fusion center a small number of pseudo-randomly obtained samples. Subsequently, the full data matrix is recovered using MC techniques.
  - MIMO-MC radars maintain the high resolution of MIMO radars, while requiring significantly fewer data to be communicated to the fusion center, thus enabling savings in communication power and bandwidth.
  - The sub-sampling of data matrix introduces new degrees of freedom for system design enabling additional interference power reduction at the radar receiver.

# Existing Spectrum Sharing Approaches

- Avoiding interference by large spatial separation;
- Dynamic spectrum access based on spectrum sensing;
- Spatial multiplexing: MIMO radar waveforms designed to eliminate the interference at the communication receiver [Khawar et al, 14].

# In this work

- We consider spectrum sharing between a matrix completion based MIMO (MIMO-MC) radar and a MIMO communication system.
- The communication waveforms are designed to minimize the interference to the radar RX while maintaining certain communication rate & using certain transmit power.
- A joint communication and radar system design is proposed to further reduce the interference.

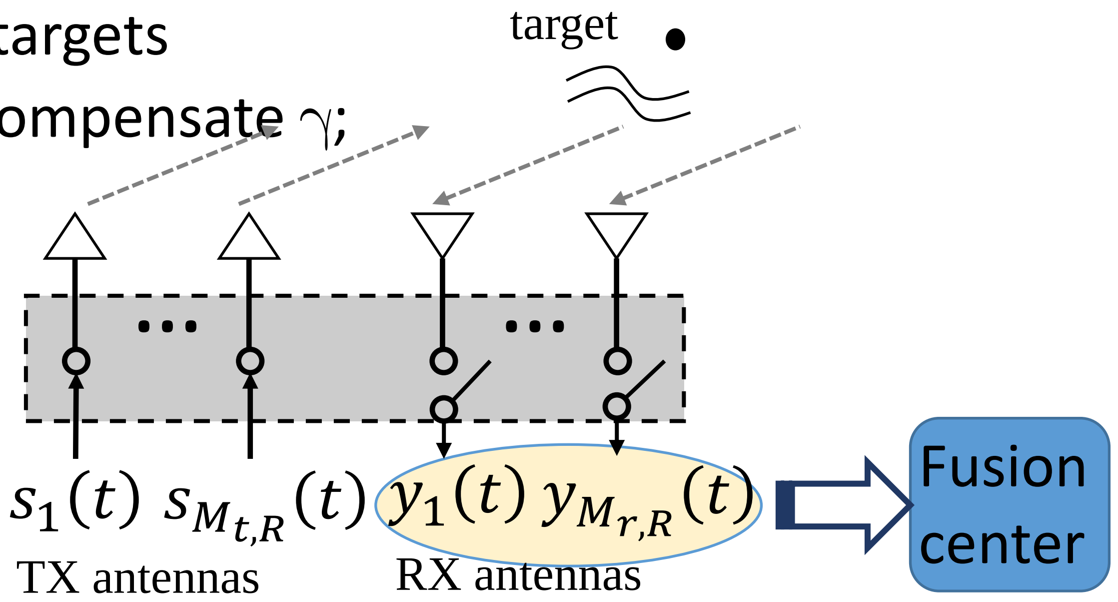
# Introduction to the Matrix Completion MIMO radar (MIMO-MC)

- The received matrix signal at the radar receivers equals

$$\mathbf{Y}_R = \gamma\rho\mathbf{B}\boldsymbol{\Sigma}\mathbf{A}^T\mathbf{S} + \mathbf{W}_R \triangleq \gamma\rho\mathbf{D}\mathbf{S} + \mathbf{W}_R$$

  - $\mathbf{A}$: $\mathbb{C}^{M_{t,R}\times K}$, $\mathbf{B}$: $\mathbb{C}^{M_{r,R}\times K}$, transmit/receive manifold matrices;
  - $\boldsymbol{\Sigma}$: $\mathbb{C}^{K\times K}$, diagonal matrix contains target reflection coefficients;
  - $\mathbf{S}$: $\mathbb{C}^{M_{t,R}\times L}$, coded MIMO radar waveforms, which are chosen orthonormal;
  - $\gamma$: path loss introduced by the far-field targets
  - $\rho$: TX power, which is large in order to compensate $\gamma$;

| Notation | |
|---|---|
| $M_{t,R}$ | # of radar TX antennas |
| $M_{r,R}$ | # of radar RX antennas |
| $K$ | # of targets |
| $L$ | Length of waveform |
| $\mathbf{W}_R$ | Additive noise |



- **DS** is low rank if $M_{r,R}$ and $L>>K$.
- Random subsampling is applied to each receive antenna. The matrix formulated at the fusion center can be expressed as:

$$\mathbf{\Omega}\circ\mathbf{Y}_R = \mathbf{\Omega}\circ(\gamma\rho\mathbf{DS}) + \mathbf{\Omega}\circ\mathbf{W}_R$$

where $\mathbf{\Omega}$ is a matrix with binary entries, whose "1"s correspond to sampling times at the RX antennas, and $\circ$ denotes Hadamard product.

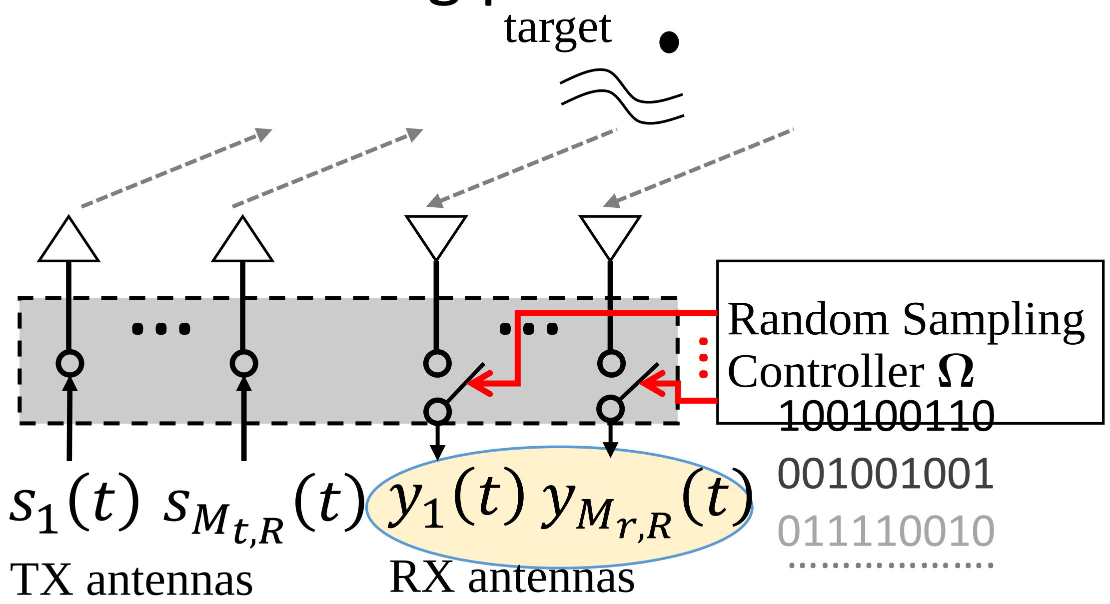
- Matrix completion can be applied to recover **DS** using partial entries of $\mathbf{Y}_R$ if [Bhojanapalli and Jain, 14]:
  - **DS** has low coherence;
  - $\mathbf{\Omega}$ has large spectral gap.

$\mathbf{\Omega}$

$M_{r,R} \times L$

Subsampling rate $p = \|\mathbf{\Omega}\|_0 / LM_{r,R}$

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

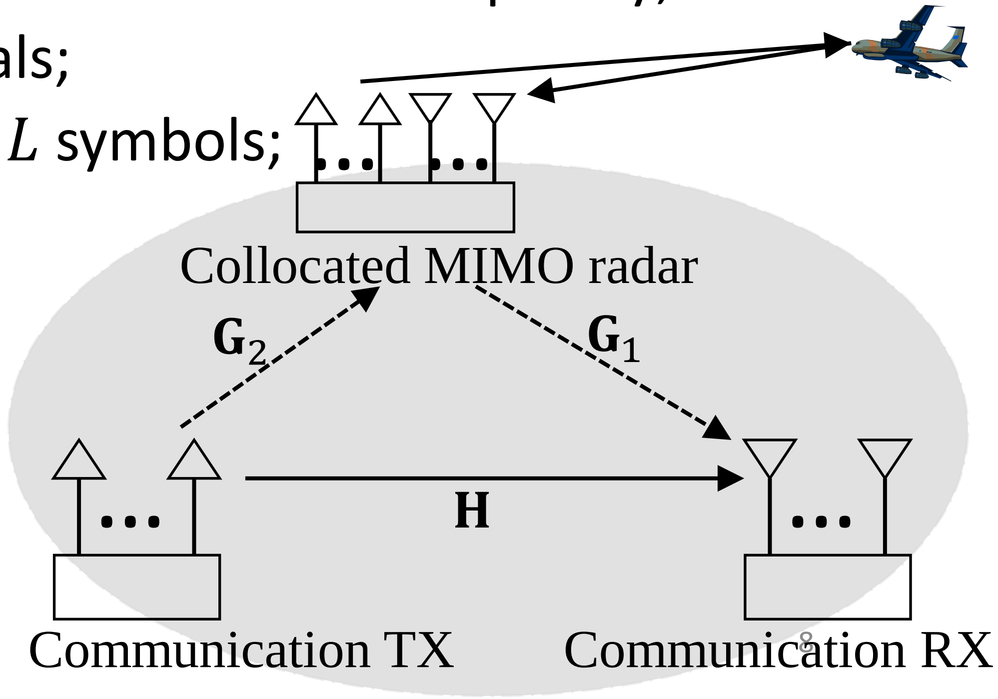


# The Coexistence Signal Model

Consider a MIMO communication system which coexists with a MIMO-MC radar system as shown below.

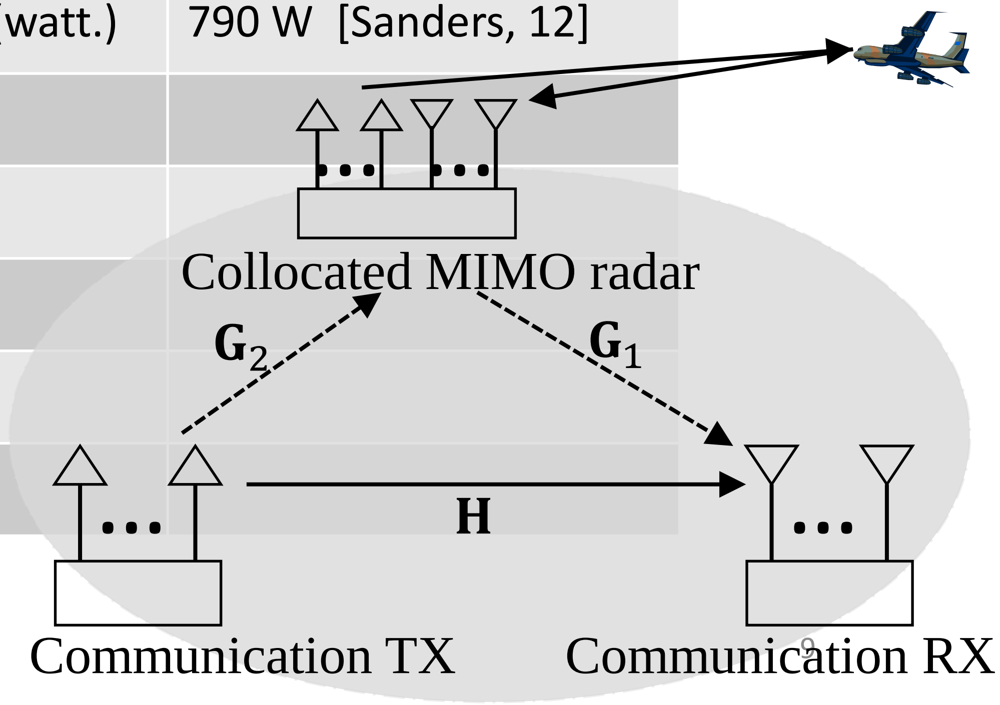
Assumptions:

- MIMO radar and communication system use the same carrier frequency;
- Flat fading, narrow band radar and comm signals;
- Block fading: the channels remain constant for $L$ symbols;
- Both systems have the same symbol rate;



# An example of system parameters

| Radar Parameters | value | Communication System | value |
|---|---|---|---|
| Carrier Freq. ($f_c$) | 3550 MHz | Carrier Freq. ($f_c$) | 3550 MHz |
| Baseband Bandwidth ($w$) | 10 MHz | Subband Bandwidth ($w$) | 5-30 MHz |
| Max Symbol rate ($f_s^R$) | 20 MHz | Max Symbol rate ($f_s^C$) | 5-30 MHz |
| Transmit power (watt.) | 750kW [Sanders, 12] | Transmit power (watt.) | 790 W [Sanders, 12] |
| Range resolution | $c/(2*f_s^R)$ = 7.5m | | |
| Pulse repetition freq. (PRF) | 20kHz | | |
| Unambiguous range | $c/(2*\text{PRF})$=7.5 km | | |
| Symbols per pulse ($L$) | 512 | | |
| Duty cycle | 50% | | |

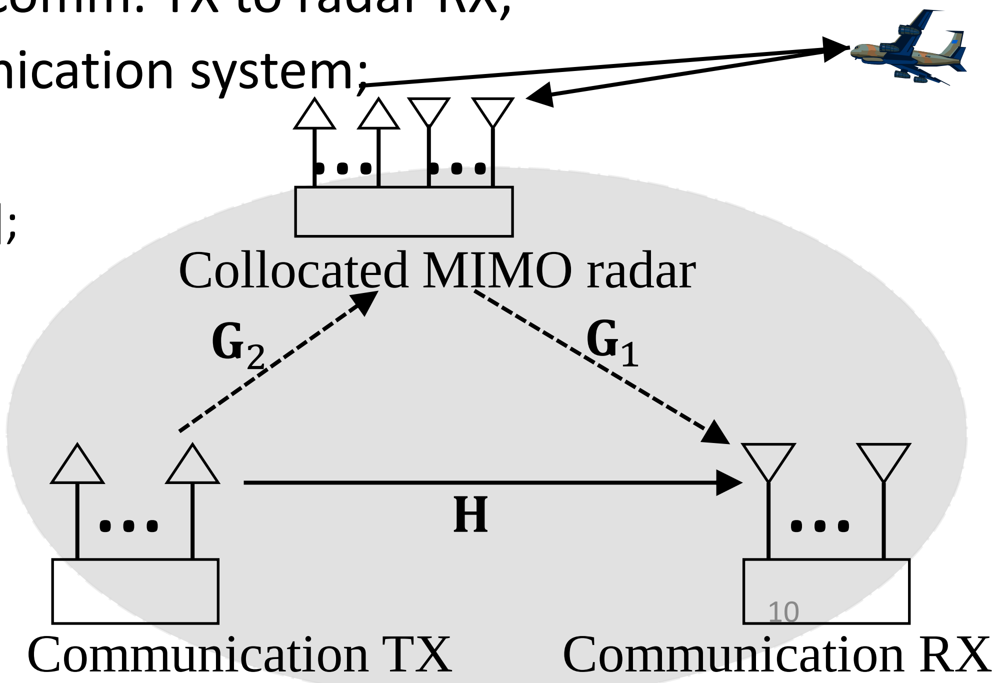


- The received signals at the MIMO-MC radar and communication RX are

$$\boldsymbol{\Omega}_l \circ \mathbf{y}_R(l) = \boldsymbol{\Omega}_l \circ \left[\gamma\rho\mathbf{D}\mathbf{s}(l) + e^{j\alpha_{2l}}\mathbf{G}_2\mathbf{x}(l) + \mathbf{w}_R(l)\right],$$
$$\mathbf{y}_C(l) = \mathbf{H}\mathbf{x}(l) + e^{j\alpha_{1l}}\mathbf{G}_1\mathbf{s}(l) + \mathbf{w}_C(l), \qquad \forall l \in \mathbb{N}_L^+,$$

where

- $l$ is the sampling time instance, $\boldsymbol{\Omega}_l$ is the $l$-th column of $\boldsymbol{\Omega}$;
- $\mathbf{H}$: $\mathbb{C}^{M_{t,C} \times M_{r,C}}$, the communication channel;
- $\mathbf{G_1}$: $\mathbb{C}^{M_{t,R} \times M_{r,C}}$, the interference channel from the radar TX to comm. RX;
- $\mathbf{G_2}$: $\mathbb{C}^{M_{t,C} \times M_{r,R}}$, the interference channel from the comm. TX to radar RX;
- $\mathbf{s}(l)$ and $\mathbf{x}(l)$: transmit vector by radar and communication system;
- $e^{j\alpha_{1l}}$ and $e^{j\alpha_{2l}}$: random phase jitters
  - Modeled as a Gaussian process in [Mudumbai et al, 07];
  - We model $\alpha_{il} \sim \mathcal{N}(0, \sigma_\alpha^2), \forall i \in \{1,2\}, \forall l \in \{1, \dots, L\}$
  - Typical value of $\sigma_\alpha^2 \approx 2.5 \times 10^{-3}$ [Razavi, 96];



- Grouping $L$ samples together, we have

$$\mathbf{\Omega}\circ\mathbf{Y}_R = \mathbf{\Omega}\circ(\gamma\rho\mathbf{D}\mathbf{S} + \mathbf{G}_2\mathbf{X}\mathbf{\Lambda}_2 + \mathbf{W}_R),$$
$$\mathbf{Y}_C = \mathbf{H}\mathbf{X} + \rho\mathbf{G}_1\mathbf{S}\mathbf{\Lambda}_1 + \mathbf{W}_C, \quad \text{where } \mathbf{\Lambda}_i = \mathrm{diag}\left(e^{j\alpha_{i1}}, \dots, e^{j\alpha_{iL}}\right), i \in \{0,1\}.$$

- Based on knowledge of radar waveforms $\mathbf{S}$ and $\mathbf{G}_1$ the communication system can reject some interference due to the radar via subtraction, but there still is some residual error due to the random phase jitters

$$\rho\mathbf{G}_1\mathbf{S}(\mathbf{\Lambda}_1 - \mathbf{I}) \approx \rho\mathbf{G}_1\mathbf{S}\mathbf{\Lambda}_\alpha, \quad \text{where } \mathbf{\Lambda}_\alpha = \mathrm{diag}(j\alpha_{11}, \dots, j\alpha_{1L}).$$

- The signal at the communication receiver after interference cancellation equals

$$\tilde{\mathbf{Y}}_c = \mathbf{H}\mathbf{X} + \rho\mathbf{G}_1\mathbf{S}\mathbf{\Lambda}_\alpha + \mathbf{W}_C.$$

- $\Lambda_\alpha$ is imaginary Gaussian. Capacity is achieved by non-circularly symmetric complex Gaussian codewords, whose covariance and complementary covariance matrix are required to be designed simultaneously.
- We consider the circularly symmetric complex Gaussian codewords $\mathbf{x}(l) \sim \mathcal{CN}(0, \mathbf{R}_{xl})$.
- The communication system aims at designing the covariance matrix $\{\mathbf{R}_{xl}\}$ to
  - Minimize its interference to MIMO-MC radars
  - While maintaining certain capacity & using certain transmit power

# Spectrum Sharing based on Optimum Communication Waveform Design

- The total TX power of the communication TX antennas equals
$$\mathbb{E}\{\mathrm{Tr}(\mathbf{X}\mathbf{X}^H)\} = \sum_{l=1}^{L} \mathrm{Tr}(\mathbf{R}_{xl}) \ .$$
- The interference plus noise covariance is given as
$$\mathbf{R}_{wl} \triangleq \rho^2 \sigma_\alpha^2 \mathbf{G}_1 \mathbf{s}(l) \mathbf{s}^H(l) \mathbf{G}_1^H + \sigma_\alpha^2 \mathbf{I}.$$
- The interference covariance changes from symbol to symbol. Thus, dynamic resource allocation need to be implemented by designing the covariance $\{\mathbf{R}_{xl}\}$.
- Similar to the definition of ergodic capacity, the achieved capacity is the average over $L$ symbols, *i.e.*,
$$\mathrm{AC}(\{\mathbf{R}_{xl}\}) \triangleq \frac{1}{L} \sum_{l=1}^{L} \log_2 |\mathbf{I} + \mathbf{R}_{wl}^{-1} \mathbf{H} \mathbf{R}_{xl} \mathbf{H}^H|.$$

# Interference to the MIMO-MC Radar

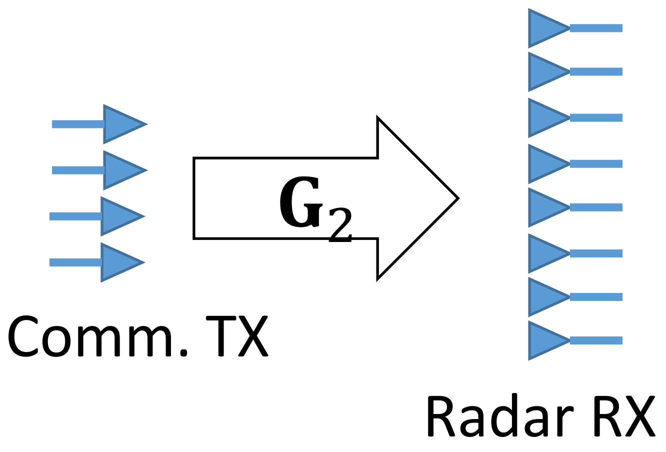
- The total interference power (TIP) exerted at the radar RX antennas equals

$$\text{TIP} \triangleq \mathbb{E}\{\text{Tr}(\mathbf{G}_2\mathbf{X}\boldsymbol{\Lambda}_2\boldsymbol{\Lambda}_2^H\mathbf{X}^H\mathbf{G}_2^H)\} = \sum_{l=1}^{L} \text{Tr}(\mathbf{G}_2\mathbf{R}_{xl}\mathbf{G}_2^H).$$

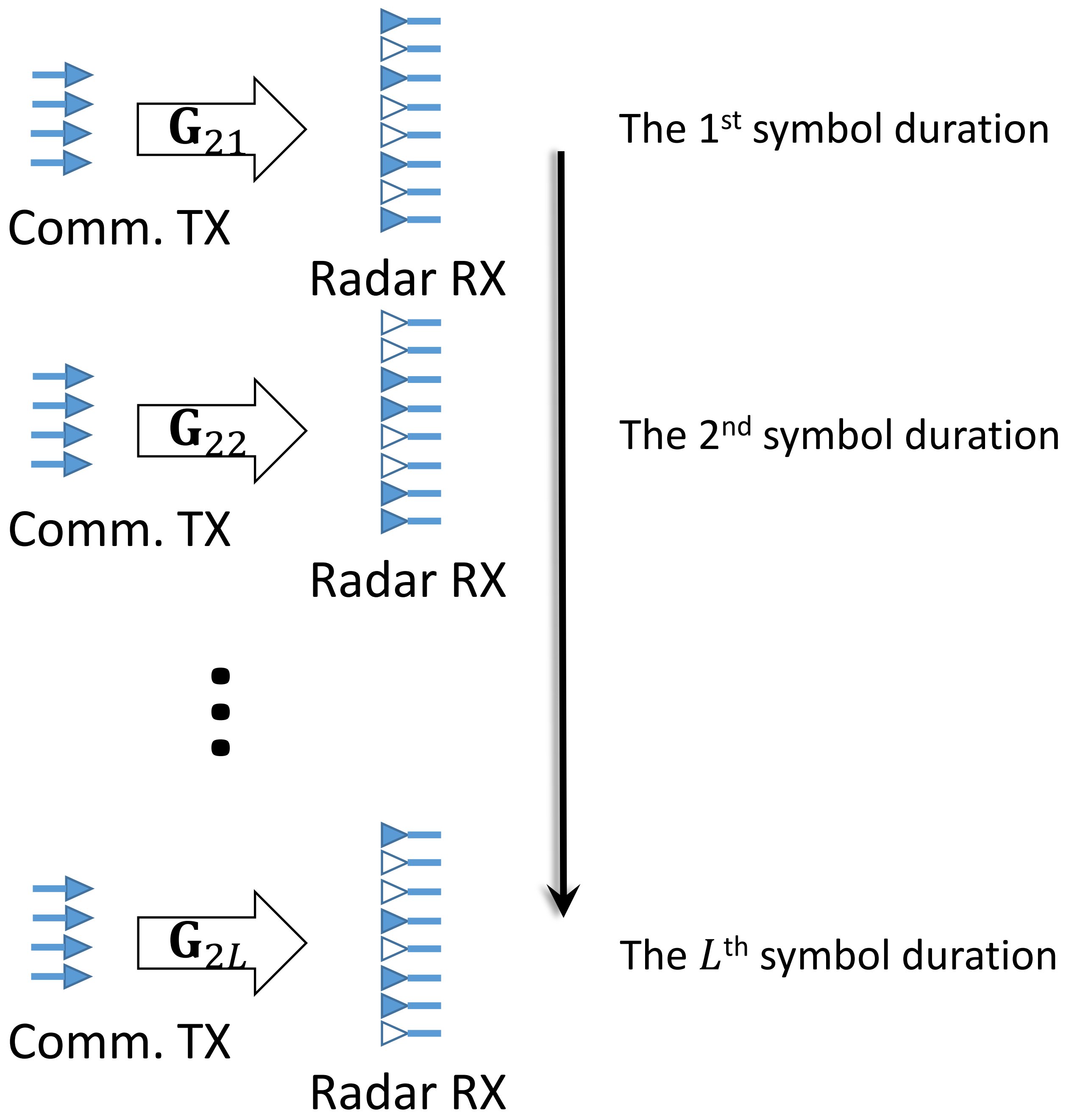
- Recall that only partial entries of $\mathbf{Y}_R$ are forwarded to the fusion center, which implies that only a portion of TIP affects the MIMO-MC radar.
- The *effective* interference power to MIMO-MC radar is given as:

$$\begin{aligned}\text{EIP} &\triangleq \mathbb{E}\{\text{Tr}(\boldsymbol{\Omega}\circ(\mathbf{G}_2\mathbf{X}\boldsymbol{\Lambda}_2)(\boldsymbol{\Omega}\circ(\mathbf{G}_2\mathbf{X}\boldsymbol{\Lambda}_2))^H)\} \\ &= \sum_{l=1}^{L} \text{Tr}\left(\mathbf{G}_{2l}\mathbf{R}_{xl}\mathbf{G}_{2l}^H\right) = \sum_{l=1}^{L} \text{Tr}(\Delta_l\mathbf{G}_2\mathbf{R}_{xl}\mathbf{G}_2^H),\end{aligned}$$

where $\mathbf{G}_{2l} = \Delta_l\mathbf{G}_2$ and $\Delta_l = \text{diag}(\boldsymbol{\Omega}_l)$.



$\mathbf{G}_2$
Comm. TX
Radar RX

$\mathbf{G}_{21}$
Comm. TX
Radar RX
The 1st symbol duration
$\mathbf{G}_{22}$
Comm. TX
Radar RX
The 2nd symbol duration
$\mathbf{G}_{2L}$
Comm. TX
Radar RX
The $L$th symbol duration

# Two Spectrum Sharing Approaches

- In the *noncooperative* approach, the communication system has no knowledge of $\boldsymbol{\Omega}$. The communication system will design its covariance matrix to minimize the TIP:

$$(\mathrm{P}_0)\ \ \min_{\{\mathbf{R}_{xl}\}} \mathrm{TIP}(\{\mathbf{R}_{xl}\}) \quad \text{s.t.} \ \sum_{l=1}^{L} \mathrm{Tr}(\mathbf{R}_{xl}) \leq P_t, \quad \mathrm{AC}(\{\mathbf{R}_{xl}\}) \geq C, \{\mathbf{R}_{xl}\} \succcurlyeq 0.$$

$$\Rightarrow \{\mathbf{R}_{xl}\} \in \mathbb{X}_0$$

- In the *cooperative* approach, the MIMO-MC radar shares its sampling scheme $\boldsymbol{\Omega}$ with the communication system. Now, the spectrum sharing problem can be formulated as:

$$(\mathrm{P}_1)\ \ \min_{\{\mathbf{R}_{xl}\}} \mathrm{EIP}(\{\mathbf{R}_{xl}\}) \quad \text{s.t.} \quad \{\mathbf{R}_{xl}\} \in \mathbb{X}_0$$

# Comparison

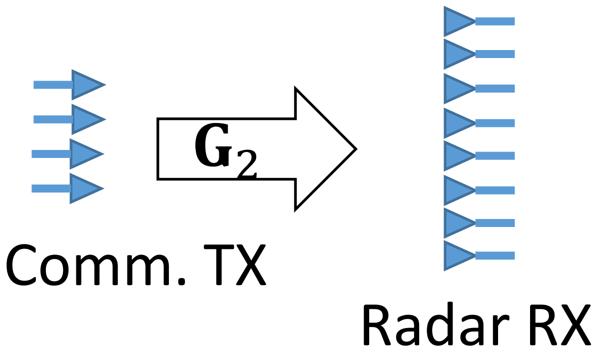
**Theorem 1**

For any $P_t$ and $C$, the EIP achieved by the cooperative approach in $(\mathbf{P}_1)$ is less or equal than that of the noncooperative approach via $(\mathbf{P}_0)$.

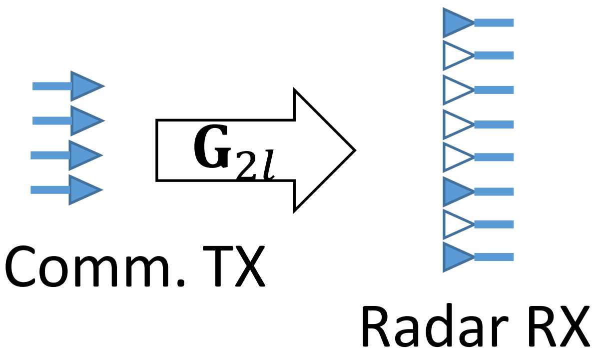
- There are certain scenarios in which the cooperative approach outperforms significantly the noncooperative one in terms of EIP.

$$\text{TIP} = \sum_{l=1}^{L} \mathrm{Tr}(\mathbf{G}_2 \mathbf{R}_{xl} \mathbf{G}_2^H)$$

All transmit directions would introduce non-zero interference.




$$\text{EIP} = \sum_{l=1}^{L} \mathrm{Tr}\left(\mathbf{G}_{2l} \mathbf{R}_{xl} \mathbf{G}_{2l}^H\right)$$




There are directions that would introduce zero EIP.

# Spectrum Sharing based on Joint Comm. and Radar System Design

- In the first two approaches, the random sampling scheme $\boldsymbol{\Omega}$ of MIMO-MC radar is predetermined.
- The joint design of $\boldsymbol{\Omega}$ and $\{\mathbf{R}_{xl}\}$ is expected to further reduce EIP

$$(\mathrm{P}_2)\ \ \{\{\mathbf{R}_{xl}\},\boldsymbol{\Omega}\} = \underset{\{\mathbf{R}_{xl}\},\boldsymbol{\Omega}}{\arg\min} \sum_{l=1}^{L} \mathrm{Tr}(\Delta_l \mathbf{G}_2 \mathbf{R}_{xl} \mathbf{G}_2^H)$$
$$\text{s.t.}\ \ \{\mathbf{R}_{xl}\} \in \mathbb{X}_0, \Delta_l = \mathrm{diag}(\boldsymbol{\Omega}_l), \boldsymbol{\Omega} \text{ is proper.}$$

$\Omega$ has binary entries;
$\Omega$ has large spectral gap;
fraction of "1"s is $p$.

- We use alternating optimization to solve $(\mathbf{P}_2)$

$$\{\mathbf{R}_{xl}^n\} = \underset{\{\mathbf{R}_{xl}\}\in\mathbb{X}_0}{\arg\min} \sum_{l=1}^{L} \mathrm{Tr}(\Delta_l^{n-1}\, \mathbf{G}_2 \mathbf{R}_{xl} \mathbf{G}_2^H) \qquad (1)$$

$$\boldsymbol{\Omega}^n = \underset{\boldsymbol{\Omega}}{\arg\min} \sum_{l=1}^{L} \mathrm{Tr}(\Delta_l \mathbf{G}_2 \mathbf{R}_{xl}^n \mathbf{G}_2^H) \qquad (2)$$
$$\text{s.t.}\ \ \Delta_l = \mathrm{diag}(\boldsymbol{\Omega}_l), \boldsymbol{\Omega} \text{ is proper.}$$

- For the problem (2), it is difficult to find an $\boldsymbol{\Omega}$ that has binary entries, a large spectral gap and a certain percentage of "1"s.
- Noticing that row and column permutations of the sampling matrix would not affect its singular values and thus the spectral gap, we propose to optimize the sampling scheme by permuting the rows and columns of an initial sampling matrix $\boldsymbol{\Omega}^0$:

$$\boldsymbol{\Omega}^n = \arg\min_{\boldsymbol{\Omega}} \mathrm{Tr}(\boldsymbol{\Omega}^T \mathbf{Q}^n) \quad \text{s.t.}\ \boldsymbol{\Omega} \in \wp(\boldsymbol{\Omega}^{n-1}), n = 1,2,\ldots \qquad (3)$$

where the $l$-th column of $\mathbf{Q}^n$ contains the diagonal entries of $\mathbf{G}_2 \mathbf{R}_{xl}^n \mathbf{G}_2^H$, $\wp(\boldsymbol{\Omega}^{n-1})$ denotes the set of matrices obtained by arbitrary row and/or column permutations.

- $\boldsymbol{\Omega}^0$ a is uniformly random sampling matrix, whose fraction of "1"s is $p$. Also, $\boldsymbol{\Omega}^0$ has large spectral gap [Bhojanapalli and Jain, 14]. Therefore, $\boldsymbol{\Omega}^n, \forall n$ is proper.
- The brute-force searching for (3) is NP hard. By alternately optimizing w.r.t. row permutation and column permutation on $\boldsymbol{\Omega}^{n-1}$, we can solve $\boldsymbol{\Omega}^n$ using a sequence of linear assignment problems.

# Mismatched symbol rates

- In the above, the waveform symbol duration of the radar system is assumed to match that of the communication system.
- However, the proposed techniques can also be applied for the mismatched cases.
- The communication system only need to know the radar sampling time instances to construct the EIP.
  - If $f_s^R < f_s^C$, the interference arrived at the radar RX will be down-sampled. The communication symbols which are not sampled would introduce zero interference power to the radar RX. Therefore, EIP only contains the communication symbols which are sampled by the radar RX.
  - If $f_s^R > f_s^C$, the interference arrived at the radar RX will be over-sampled. One individual communication symbol will introduce interference to the radar system in $\lfloor f_s^R / f_s^C \rfloor$ consecutive sampling time instances. Correspondingly, in the expression of the EIP, each individual communication transmit covariance matrix will be weighted by the sum of interference channels for $\lfloor f_s^R / f_s^C \rfloor$ radar sampling time instances.

# Simulations

- MIMO-MC radar with half-wavelength uniform linear TX&RX arrays transmit Gaussian orthonormal waveforms. One target at angle $30°$ and with reflection coefficient $0.2+0.1j$.
- $\mathbf{H}$ is with entries distributed as $\mathcal{CN}(0,1)$; $\mathbf{G}_1$ and $\mathbf{G}_2$ are with entries distributed as $\mathcal{CN}(0,0.1)$.
- $L = 32, \sigma_C^2 = .01, \gamma^2 = -30$dB, $\rho^2 = 1000\, L/M_{t,R}\, , \sigma_\alpha^2 = 10^{-3}$.
- Four different realizations of $\mathbf{\Omega}^0$ are evaluate for all the proposed algorithms.
- The obtained $\mathbf{R}_{xl}$ is used to generate $x(l) \; = \mathbf{R}_{xl}^{1/2}$randn$(M_{t,C}, 1)$.
- The TFOCUS package is used for matrix completion at the radar fusion center.
- EIP and MC relative recovery error ($\left\|\mathbf{DS} - \widehat{\mathbf{DS}}\right\|_F / \|\mathbf{DS}\|_F$) are used as the performance metrics.
- For comparison, we also implement a "selfish communication" scenario, where the communication system minimizes the TX power to achieve certain average capacity without any concern about the interferences it exerts to the radar system.

# Simulations

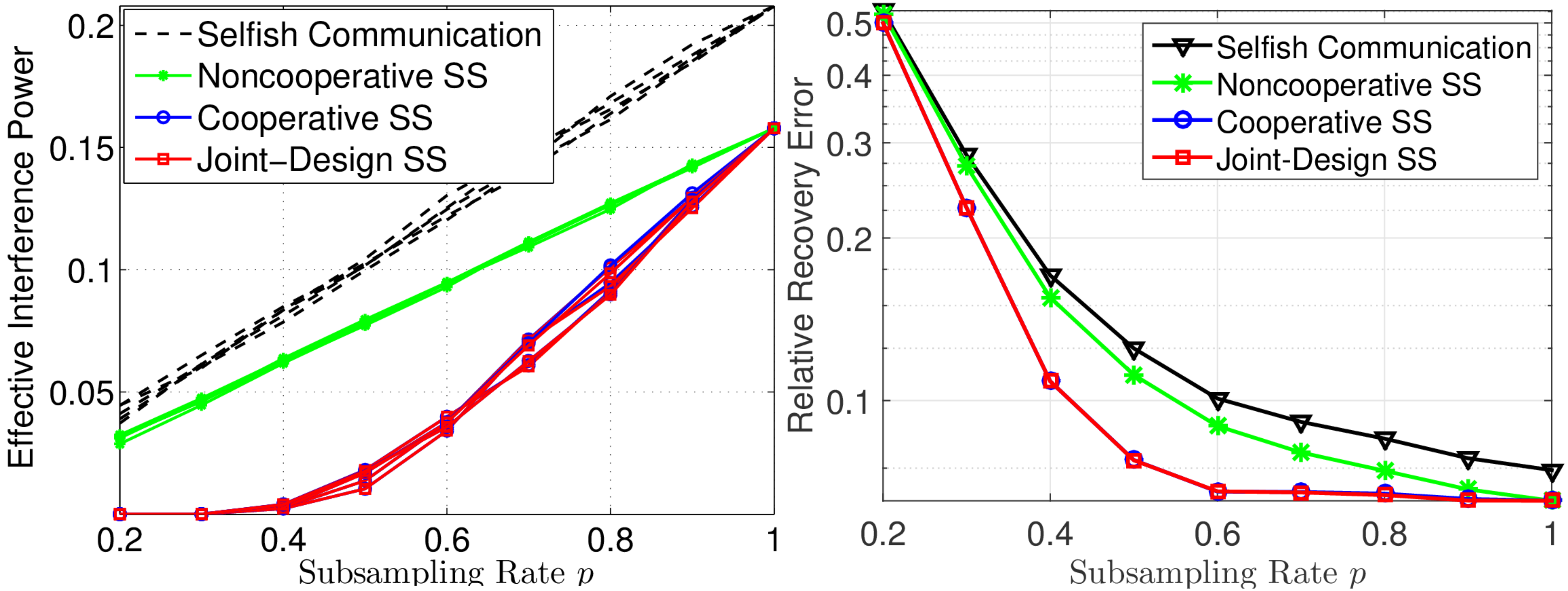



Figure. Spectrum sharing under different sub-sampling rates.
$M_{t,R} = 4, M_{r,R} = 8, M_{t,C} = 8, M_{r,C} = 4,$
$P_t = L, C = 12$bits/symbol, SNR=25dB

# Simulations

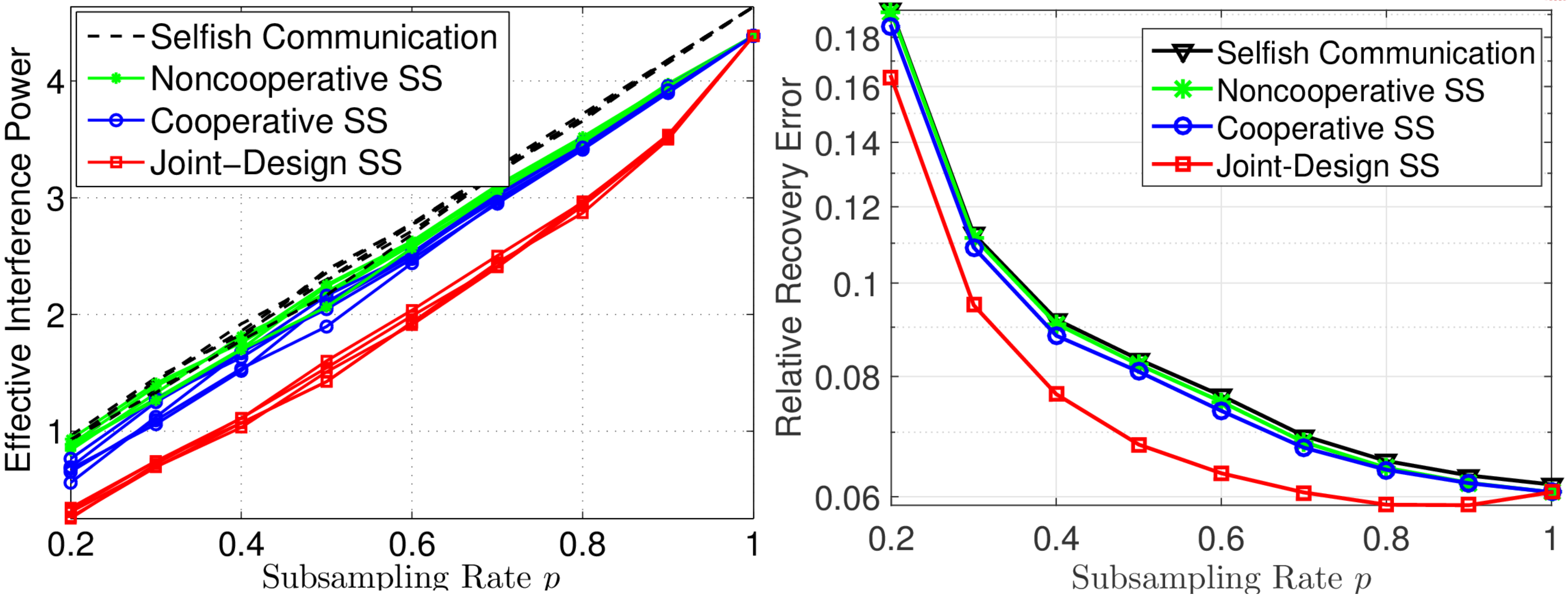



Figure. Spectrum sharing under different sub-sampling rates.
$M_{t,R} = 16, M_{r,R} = 32, M_{t,C} = 4, M_{r,C} = 4,$
$P_t = L, C =$ 12bits/symbol, SNR=25dB

# Conclusions

- We have considered spectrum sharing between a MIMO communication system and a MIMO-MC radar system.
- We have proposed three strategies to reduce the interference from the communication TX to the radar.
- By appropriately designing the communication system waveforms, the interference can be greatly reduced.
- The joint design of the communication waveforms and the sampling scheme at the radar RX antennas can lead to further reduction of the interference.
- In future work, we will consider the spectrum sharing problem for MIMO-MC radar model, where targets are distributed across different range bins.

# References

A Khawar, A Abdel-Hadi, and T.C. Clancy, “Spectrum sharing between s-band radar and lte cellular system: A spatial approach,” in 2014 IEEE International Symposium on Dynamic Spectrum Access Networks, April 2014, pp. 7–14.

Shunqiao Sun and Athina Petropulu, “MIMO-MC Radar: A MIMO Radar Approach Based on Matrix Completion”, submitted to IEEE TAES, 2014.

S. Bhojanapalli and P. Jain, “Universal matrix completion,” arXiv preprint arXiv:1402.2324, 2014

R. Mudumbai, G. Barriac, and U. Madhow, “On the feasibility of distributed beamforming in wireless networks,” IEEE Transactions on Wireless Communications, vol. 6, no. 5, pp. 1754–1763, 2007.

B. Razavi, “A study of phase noise in CMOS oscillators,” IEEE Journal of Solid-State Circuits, vol. 31, no. 3, pp. 331–343, 1996.

F. H. Sanders, R. L. Sole, J. E. Carroll, G. S. Secrest, and T. L. Allmon, “Analysis and resolution of RF interference to radars operating in the band 2700–2900 MHz from broadband communication transmitters,” US Dept. of Commerce, Tech. Rep. NTIA Technical Report TR-13-490,2012.

# Thank you!