Radar Precoding for Spectrum Sharing Between Matrix Completion Based MIMO Radars and A MIMO Communication System

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Dec. 15, 2015

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Work supported by NSF under Grant ECCS-1408437

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- Background on Matrix Completion based MIMO (MIMO-MC) Radars
- The coexistence signal model
- The proposed spectrum sharing scheme
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Motivation

- Radar and communication systems overlap in the spectrum domain thus causing interference to each other.
- Spectrum sharing can increase spectrum efficiency.

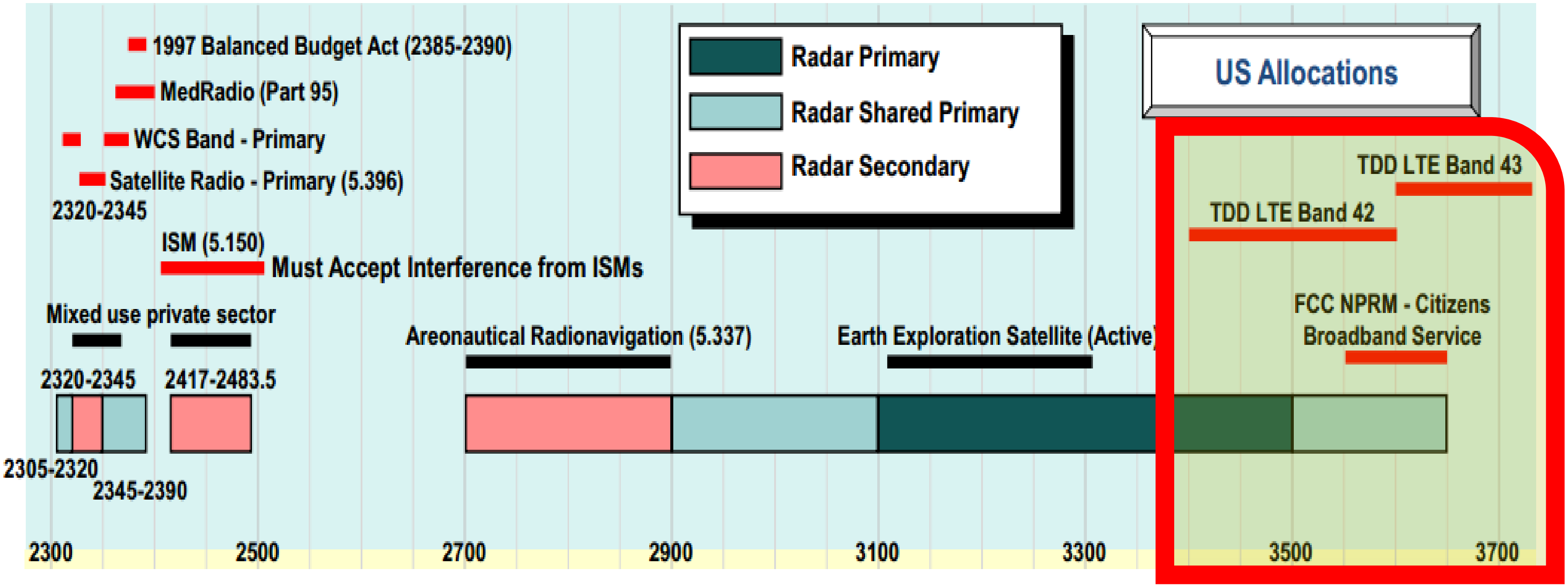


Figure from DARPA Shared Spectrum Access for Radar and Communications (SSPARC)

Background

- Matrix completion based MIMO radars (MIMO-MC) [1] is a good candidate for reducing interference at the radar receiver [2].
  - Traditional MIMO radars transmit orthogonal waveforms from their transmit (TX) antennas, and their receive (RX) antennas forward their measurements to a fusion center to populate a "data matrix" for further processing.
  - Based on the low-rankness of the data matrix, MIMO-MC radar RX antennas forward to the fusion center a small number of pseudo-randomly obtained samples. Subsequently, the full data matrix is recovered using MC techniques.
  - MIMO-MC radars maintain the high resolution of MIMO radars, while requiring significantly fewer data to be communicated to the fusion center, thus enabling savings in communication power and bandwidth.
  - The sub-sampling at the RX antennas introduces new degrees of freedom for system design enabling additional interference power reduction at the radar receiver [2].

[1] S. Sun, W. U. Bajwa, and A. P. Petropulu, IEEE TAES 2015.
[2] B. Li and A. Petropulu, IEEE ICASSP 2015.

- Existing Spectrum Sharing Approaches
  - Avoiding interference by large spatial separation;
  - Dynamic spectrum access based on spectrum sensing;
  - Spatial multiplexing: MIMO radar waveforms designed to eliminate the interference at the communication receiver [1].
- Our previous work [2]

  Spectrum sharing between a MIMO-MC radar and a MIMO communication system is achieved by
  - Sharing the radar waveforms with the communication system, and
  - Jointly designing the communication system signals and the radar system sampling scheme.
- In this work

  A new framework for spectrum sharing between a MIMO-MC radar and a MIMO communication system is proposed
  - Radar precoding is jointly designed with the communication codewords to maximized the radar SINR while meeting certain rate and power constraints at the communication system.
  - Only the radar precoder is shared with the communication system, rather than the radar waveforms, which preserves the radar waveform confidentiality.

[1] A. Khawar, A Abdel-Hadi, and T.C. Clancy, IEEE DySPAN 2014.
[2] B. Li and A. Petropulu, IEEE ICASSP 2015.

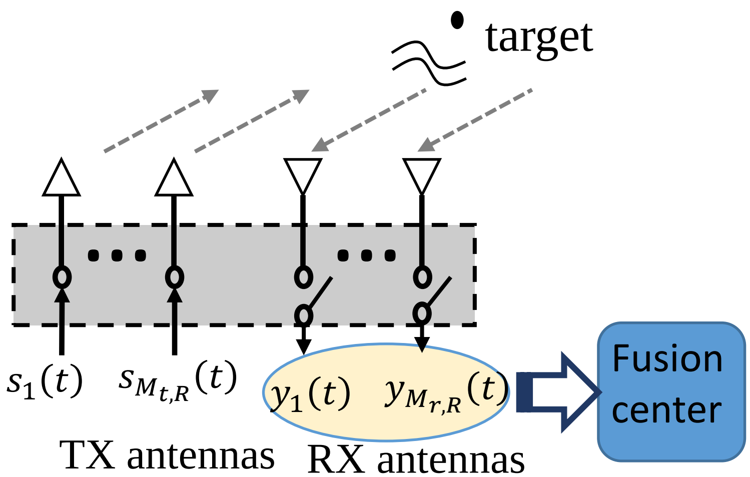
Introduction to Matrix Completion MIMO Radars (MIMO-MC)

- The received data at the radar receivers can be expressed as

$$\mathbf{Y}_R = \mathbf{B}\boldsymbol{\Sigma}\mathbf{A}^T\mathbf{P}\mathbf{S} + \mathbf{W}_R \triangleq \mathbf{D}\mathbf{P}\mathbf{S} + \mathbf{W}_R$$

  - $\mathbf{A}$: $\mathbb{C}^{M_{t,R}\times K}$, $\mathbf{B}$: $\mathbb{C}^{M_{r,R}\times K}$, transmit/receive manifold matrices;
  - $\boldsymbol{\Sigma}$: $\mathbb{C}^{K\times K}$, diagonal matrix contains target reflection coefficients;
  - $\mathbf{S}$: $\mathbb{C}^{M_{t,R}\times L}$, coded MIMO radar waveforms, taken to be orthonormal;
  - $\mathbf{P}$: $\mathbb{C}^{M_{t,R}\times M_{t,R}}$, the radar precoding matrix; $\mathbf{D} \triangleq \mathbf{B}\boldsymbol{\Sigma}\mathbf{A}^T$;

Notation	
$M_{t,R}$	# of radar TX antennas
$M_{r,R}$	# of radar RX antennas
K	# of targets
L	Length of waveform
$\mathbf{W}_R$	Additive noise



- Matrix $\mathbf{DPS}$ is low rank if $M_{r,R}$ and $L >> K$.
- Random subsampling is applied to each receive antenna. The matrix formulated at the fusion center can be expressed as:

$$\mathbf{\Omega} \circ \mathbf{Y}_R = \mathbf{\Omega} \circ (\mathbf{DPS}) + \mathbf{\Omega} \circ \mathbf{W}_R$$

where $\mathbf{\Omega}$ is a matrix with binary entries, whose "1"s correspond to sampling times at the RX antennas, and $\circ$ denotes Hadamard product.

- Matrix completion can be applied to recover $\mathbf{DPS}$ using partial entries of $\mathbf{Y}_R$ if:
  - $\mathbf{DPS}$ has low coherence;
  - $\mathbf{\Omega}$ has large spectral gap.

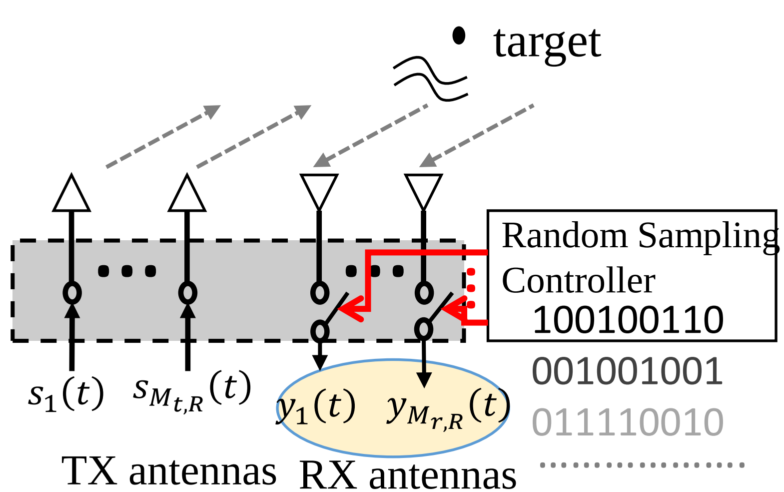
$\mathbf{\Omega}$

$M_{r,R} \times L$

Subsampling rate

$p = \|\mathbf{\Omega}\|_0 / LM_{r,R}$

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

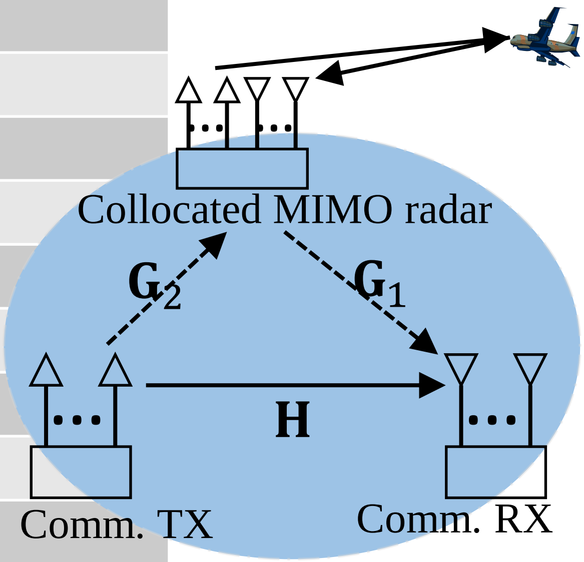


The Coexistence Signal Model

- Consider a MIMO communication system which coexists with a MIMO-MC radar system using the same carrier frequency.

- Assumptions:
  - Flat fading, narrow band radar and comm. signals;
  - Block fading: the channels remain constant for L symbols;
  - Both systems have the same symbol rate;

Parameters	Radar System	Communication System
Carrier Freq. (f_c)	3550 MHz	3550 MHz
Baseband Bandwidth (w)	0.5 MHz	0.5 MHz [3]
Sub-pulse/Symbol duration (T_b)	2 *us*	2 *us*
Transmit power	750kW [1]	790 W [1]
Range resolution	$c/(2{*}w)$ = 300m [2]	
Pulse repetition freq. (PRF)	1 kHz	
Unambiguous range	$c/(2{*}\text{PRF})$= 150 km	
Symbols per pulse (L)	128	
Duty cycle	25%	



[1] F. H. Sanders, et al, NTIA Tech. Rep TR-13-490, 2012.
[2] "Radar performance," Radtec Engineering Inc., [online] (Accessed:July 2015).
[3] Telesystem Innovations, LTE White paper, 2010.

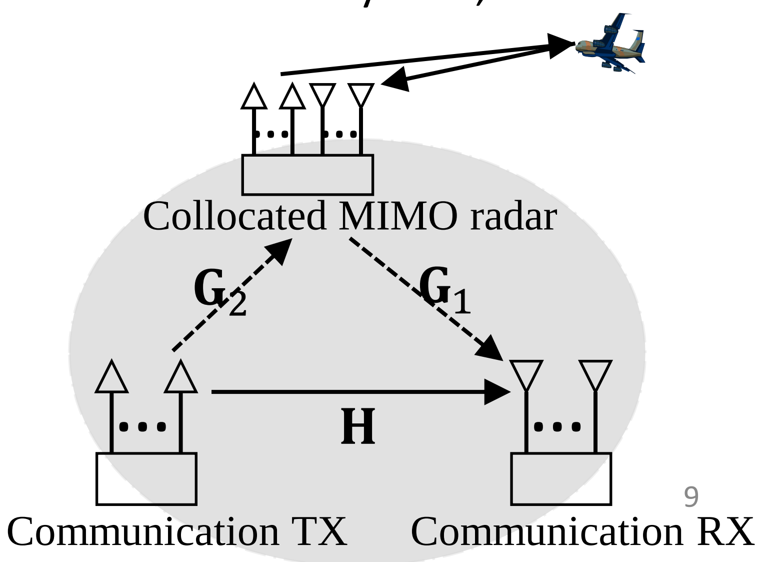
- The received signals at the MIMO-MC radar and communication RX are

$$\mathbf{\Omega}_l \circ \mathbf{y}_R(l) = \mathbf{\Omega}_l \circ \left[\mathbf{DPs}(l) + e^{j\alpha_{2l}}\mathbf{G}_2\mathbf{x}(l) + \mathbf{w}_R(l)\right],$$

$$\mathbf{y}_C(l) = \mathbf{Hx}(l) + e^{j\alpha_{1l}}\,\mathbf{G}_1\mathbf{Ps}(l) + \mathbf{w}_C(l), \forall l \in \mathbb{N}_L^+,$$

where

- l is the sampling time instance, $\mathbf{\Omega}_l$ is the l-th column of $\mathbf{\Omega}$;
- $\mathbf{H}$: $\mathbb{C}^{M_{t,C} \times M_{r,C}}$, the communication channel;
- $\mathbf{G_1}$: $\mathbb{C}^{M_{t,R} \times M_{r,C}}$, the interference channel from the radar TX to comm. RX;
- $\mathbf{G_2}$: $\mathbb{C}^{M_{t,C} \times M_{r,R}}$, the interference channel from the comm. TX to radar RX;
- $\mathbf{s}(l)$ and $\mathbf{x}(l)$: transmit vector by radar and communication system;
- $e^{j\alpha_{1l}}$ and $e^{j\alpha_{2l}}$: random phase jitters
  - We do not make any assumption on α_{il}.



- Grouping L samples together, we have

$$\boldsymbol{\Omega} \circ \mathbf{Y}_R = \boldsymbol{\Omega} \circ (\mathbf{DPS} + \mathbf{G}_2\mathbf{X}\boldsymbol{\Lambda}_2 + \mathbf{W}_R),$$

$$\mathbf{Y}_C = \mathbf{HX} + \mathbf{G}_1\mathbf{PS}\boldsymbol{\Lambda}_1 + \mathbf{W}_C,$$

where $\boldsymbol{\Lambda}_i = \mathrm{diag}\left(e^{j\alpha_{i1}}, \ldots, e^{j\alpha_{iL}}\right), i \in \{0,1\}$.

- We consider radar precoding; the precoding matrix $\mathbf{P}$ is employed and shared with the communication system
  - We take $\mathbf{S}$ to be a random orthonormal matrix;
  - Communication codewords are circularly symmetric complex Gaussian $\mathbf{x}(l) \sim \mathcal{CN}(0, \mathbf{R}_x), \forall l$;
- Matrices $\mathbf{P}$ and $\mathbf{R}_x$ are jointly designed to
  - maximize the SINR at the MIMO-MC radar receiver, while maintaining certain communication system rate and power constraints.

The Proposed Spectrum Sharing Method

- For the MIMO communication system:
  - The total TX power of the communication TX antennas equals

$$\mathbb{E}\{\mathrm{Tr}(\mathbf{X}\mathbf{X}^H)\} = L\mathrm{Tr}(\mathbf{R}_x).$$

  - The interference plus noise covariance is given as

$$\mathbf{R}_w \triangleq \mathbf{G}_1\mathbf{P}\mathbb{E}\{\mathbf{s}(l)\mathbf{s}^H(l)\}\mathbf{P}^H\mathbf{G}_1^H + \sigma_C^2\mathbf{I}$$

$$= \mathbf{G}_1\mathbf{\Phi}\mathbf{G}_1^H + \sigma_C^2\mathbf{I},$$

    where $\mathbf{\Phi} = \mathbf{P}\mathbf{P}^H/L$. The second equality holds because the entries of $\mathbf{S}$ can be approximated by i.i.d. Gaussian random variables with distribution $\mathcal{N}(0, 1/L)$, if $M_{t,R} = \mathcal{O}(L/\mathrm{ln}L)$ [1].
  - The communication rate achieved, also a lower bound of the channel capacity, is given by

$$\underline{\mathrm{C}}(\mathbf{R}_x, \mathbf{\Phi}) \triangleq \log_2|\mathbf{I} + \mathbf{R}_w^{-1}\mathbf{H}\mathbf{R}_x\mathbf{H}^H|.$$

[1] T. Jiang, Annals Prob. 2006.

- For the MIMO-MC radar:
  - The total interference power (TIP) exerted at the RX antennas equals

$$\mathrm{TIP} \triangleq \mathbb{E}\{\mathrm{Tr}(\mathbf{G}_2\mathbf{X}\mathbf{\Lambda}_2\mathbf{\Lambda}_2^H\mathbf{X}^H\mathbf{G}_2^H)\} = L\mathrm{Tr}(\mathbf{G}_2\mathbf{R}_x\mathbf{G}_2^H).$$

  - Recall that only partial entries of $\mathbf{Y}_R$ are forwarded to the fusion center, which implies that only a portion of TIP affects the MIMO-MC radar.
  - The *effective* interference power (EIP) to MIMO-MC radar is given as:

$$\mathrm{EIP} \triangleq \mathbb{E}\left\{\mathrm{Tr}\left(\mathbf{\Omega} \circ (\mathbf{G}_2\mathbf{X}\mathbf{\Lambda}_2)\left(\mathbf{\Omega} \circ (\mathbf{G}_2\mathbf{X}\mathbf{\Lambda}_2)\right)^H\right)\right\} = \mathrm{Tr}(\Delta\mathbf{G}_2\mathbf{R}_x\mathbf{G}_2^H)$$

  where $\Delta \triangleq \sum_{l=1}^{L}\Delta_l$ and $\Delta_l = \mathrm{diag}(\mathbf{\Omega}_l)$. We note that the EIP is a re-weighted version of the TIP.
  - Similarly, the effective signal power (ESP) of the target echo signal forwarded to the fusion center equals

$$\mathrm{ESP} \triangleq \mathrm{Tr}(\Delta\mathbf{D}\mathbf{\Phi}\mathbf{D}^H)$$

  - We assume that $\mathbf{D}$ is given a *priori*. In practice, such information could be obtained in various ways, e.g., in tracking applications, the parameter estimates obtained from previous tracking cycles.

- If the MIMO-MC radar shares its random sampling scheme with the communication system, the spectrum sharing problem can be formulated as:

$$(\mathbf{P}_1)\ \ \max_{\mathbf{R}_x,\mathbf{\Phi}} \text{ESINR} \equiv \frac{\text{Tr}(\Delta\mathbf{D}\mathbf{\Phi}\mathbf{D}^H)}{\text{Tr}(\Delta\mathbf{G}_2\mathbf{R}_x\mathbf{G}_2^H)+pP_{wR}}$$

$$\text{s.t. } L\text{Tr}(\mathbf{R}_x) \le P_C,\ L\text{Tr}(\mathbf{\Phi}) \le P_R,$$

$$\underline{\text{C}}(\mathbf{R}_x,\mathbf{\Phi}) \ge C, \mathbf{R}_x \succcurlyeq 0, \mathbf{\Phi} \succcurlyeq 0.$$

where $P_{wR} = LM_{r,R}\sigma_R^2$.

- Problem (P_1) is non-convex w.r.t. both $\mathbf{R}_x$ and $\mathbf{\Phi}$. A solution can be obtained via alternating optimization.
- Fixing $\mathbf{\Phi}$, the $\mathbf{R}_x$ sub-problem is given as

$$(\mathbf{P}_\text{R})\ \ \min_{\mathbf{R}_x \succcurlyeq 0}\text{Tr}(\mathbf{G}_2^H\Delta\mathbf{G}_2\mathbf{R}_x)$$

$$\text{s.t. } \underline{\text{C}}(\mathbf{R}_x,\mathbf{\Phi}) \ge C, L\text{Tr}(\mathbf{R}_x) \le P_C$$

  - The above problem is convex w.r.t. $\mathbf{R}_x$ and can be solved efficiently using the interior point method.

- Fixing $\mathbf{R}_x$, the $\boldsymbol{\Phi}$ sub-problem is given as

$$(\mathrm{P}_\Phi)\ \ \min_{\boldsymbol{\Phi}\succcurlyeq 0}\mathrm{Tr}(\mathbf{D}^H\Delta\mathbf{D}\boldsymbol{\Phi})$$
$$\text{s.t. } \underline{C}(\mathbf{R}_x,\boldsymbol{\Phi}) \geq C,\ L\mathrm{Tr}(\boldsymbol{\Phi}) \leq P_R$$

  - We can express $\underline{C}(\mathbf{R}_x,\boldsymbol{\Phi})$ as follows:

$$\underline{C}(\mathbf{R}_x,\boldsymbol{\Phi}) = \log_2|\mathbf{G}_1\boldsymbol{\Phi}\mathbf{G}_1^H + \widetilde{\mathbf{R}}_x| - \log_2|\mathbf{G}_1\boldsymbol{\Phi}\mathbf{G}_1^H + \sigma_C^2\mathbf{I}|,$$

  where $\widetilde{\mathbf{R}}_x \triangleq \sigma_C^2\mathbf{I} + \mathbf{H}\mathbf{R}_x\mathbf{H}^H$. $\underline{C}(\mathbf{R}_x,\boldsymbol{\Phi}) \geq C$ is a non-convex constraint.
  - To overcome the non-convexity, an auxiliary $\boldsymbol{\Psi}$ is introduced by transforming (P_Φ) into the following problem:

$$(\mathbf{P}_{\Phi\Psi})\ \ \max_{\boldsymbol{\Phi}\succeq 0}\mathrm{Tr}(\boldsymbol{\Delta}\mathbf{D}\boldsymbol{\Phi}\mathbf{D}^H),\ \ \text{s.t.}\ LTr(\boldsymbol{\Phi}) \leq P_R,$$
$$\log_2\left|\mathbf{G}_1\boldsymbol{\Phi}\mathbf{G}_1^H + \widetilde{\mathbf{R}}_x\right| + \max_{\boldsymbol{\Psi}\succeq 0}\log_2|\boldsymbol{\Psi}|$$
$$-\mathrm{Tr}((\mathbf{G}_1\boldsymbol{\Phi}\mathbf{G}_1^H + \sigma_C^2\mathbf{I})\boldsymbol{\Psi}) + M_{r,C} \geq C$$

  - Again, alternating optimization is applied as an inner iteration. During the n-th outer alternating iteration, let $(\boldsymbol{\Phi}^{nk},\boldsymbol{\Psi}^{nk})$ be the variables at the k-th inner iteration.

- One inner iteration is given as follows

$$\mathbf{\Psi}^{nk} = (\mathbf{G}_1 \mathbf{\Phi}^{n(k-1)} \mathbf{G}_1^H + \sigma_C^2 \mathbf{I})^{-1}$$

$$(\mathbf{P}'_{\Phi}) \quad \mathbf{\Phi}^{nk} = \arg\max_{\mathbf{\Phi} \succeq 0} \mathrm{Tr}(\mathbf{\Delta} \mathbf{D} \mathbf{\Phi} \mathbf{D}^H), \quad \mathrm{s.t.}\, L\mathrm{Tr}(\mathbf{\Phi}) \leq P_R,$$

$$\log_2 |\mathbf{I} + \mathbf{G}_1^H (\widetilde{\mathbf{R}}_x)^{-1} \mathbf{G}_1 \mathbf{\Phi}| - \mathrm{Tr}\left(\mathbf{G}_1^H \mathbf{\Psi}^{nk} \mathbf{G}_1 \mathbf{\Phi}\right) \geq C',$$

where C' is a constant w.r.t. $\mathbf{\Phi}$. $(\mathbf{P}'_{\Phi})$ is convex and can be solved efficiently.

- The proposed spectrum sharing algorithm can be summarized as

Algorithm 1 The proposed spectrum sharing method.

Input: $\mathbf{D}, \mathbf{H}, \mathbf{G}_1, \mathbf{G}_2, \mathbf{\Omega}, P_R, P_C, C, \sigma_C^2, \delta_1, \delta_2$
Initialization: $\mathbf{\Phi}^0 = P_R / M_{t,R} \mathbf{I}$
repeat
  $\mathbf{R}_x^n \leftarrow$ Solve problem $(\mathbf{P_R})$ using interior point method or $(\mathbf{P_R}$-D) using [13, **Algorithm** 1] with fixed $\mathbf{\Phi}^{n-1}$;
  $\mathbf{\Phi}^{nk} \leftarrow \mathbf{\Phi}^{n-1}$ for $k = 0$;
  repeat
    $\mathbf{\Psi}^{nk} = \left(\mathbf{G}_1 \mathbf{\Phi}^{n(k-1)} \mathbf{G}_1^H + \sigma_C^2 \mathbf{I}\right)^{-1}$;
    $\mathbf{\Phi}^{nk} \leftarrow$ Solve problem $(\mathbf{P}'_{\Phi})$ using interior point method or $(\mathbf{P}'_{\Phi}$-D) with fixed $\mathbf{\Psi}^{nk}$ and $\mathbf{R}_x^n$;
    $k \leftarrow k + 1$
  until $|\mathrm{ESP}^k - \mathrm{ESP}^{k-1}| < \delta_1$
  $\mathbf{\Phi}^n \leftarrow \mathbf{\Phi}^{nk}$
  $n \leftarrow n + 1$
until $|\mathrm{ESINR}^n - \mathrm{ESINR}^{n-1}| < \delta_2$
Output: $\mathbf{R}_x = \mathbf{R}_x^n, \mathbf{P} = \sqrt{L}(\mathbf{\Phi}^n)^{1/2}$

Simulations

- MIMO-MC radar with half-wavelength uniform linear TX&RX arrays transmit random orthonormal waveforms. Two far-field targets at angles ±60°.
- Entries in $\mathbf{H}$ are i.i.d. and $\mathbf{H}_{ij} \sim \mathcal{CN}(0,1)$; Entries in $\mathbf{G}_1$ and $\mathbf{G}_2$ are i.i.d. $\mathcal{CN}(0,0.01)$.
- $L = 32, \sigma_R^2 = \sigma_C^2 = .01, C = 20$bits/symbol, $P_C = LM_{t,C}$ (the power is normalized by the power of radar waveform).
- The obtained $\mathbf{R}_x$ is used to generate $x(l) = \mathbf{R}_x^{1/2}$randn$(M_{t,C}, 1)$.
- The TFOCUS package is used for matrix completion at the radar fusion center.
- ESINR and MC relative recovery error ($\left\|\mathbf{DPS} - \widehat{\mathbf{DPS}}\right\|_F / \|\mathbf{DPS}\|_F$) are used as the performance metrics.
- Comparing methods include
  - Method #1: no radar precoding, i.e., $\mathbf{P} = \sqrt{LP_R/M_{t,R}}\mathbf{I}$, plus "selfish communication", where the communication system minimizes the TX power to achieve certain rate without any concern about the interferences it exerts to the radar system.
  - Method #2: no radar precoding, but $\mathbf{R}_x$ being designed to minimize the interferences it exerts to the radar system while achieving certain communication rate.
  - Method #3: our previous approach where $\mathbf{S}$ is shared with the communication receiver, and there is no radar precoding.

Simulations

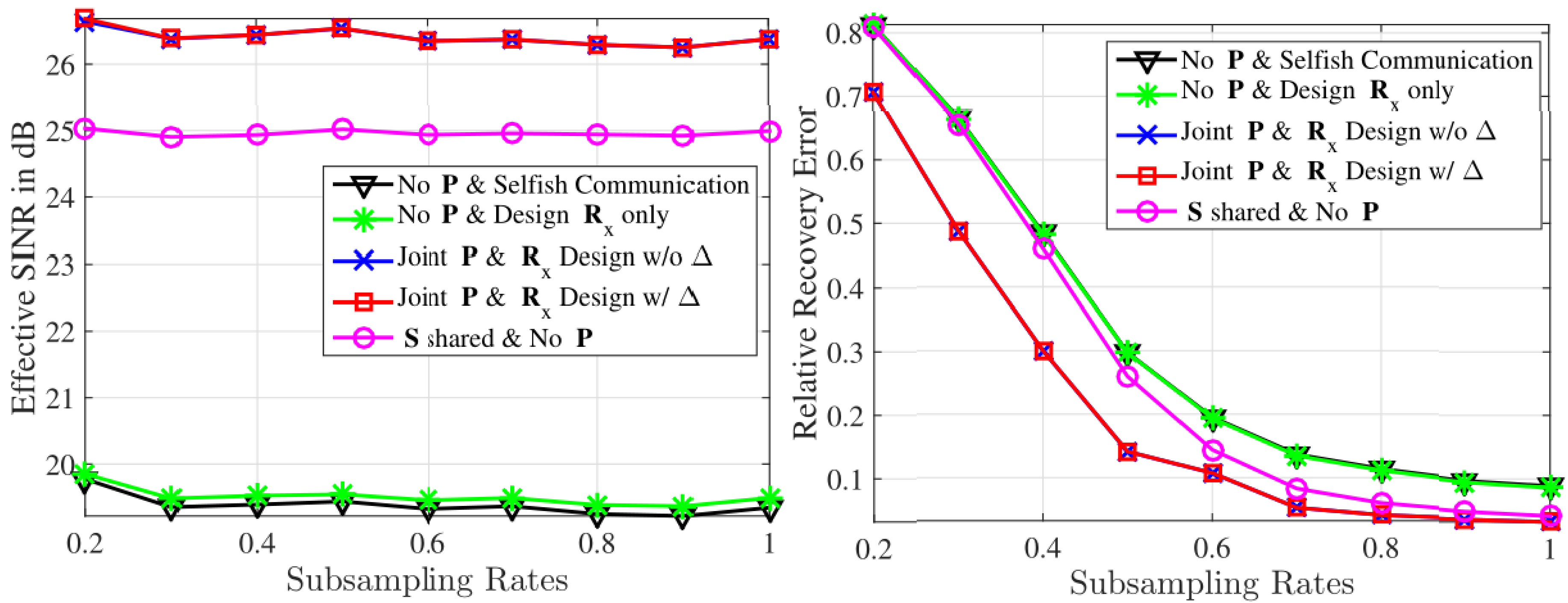


Figure 1. Spectrum sharing under different sub-sampling rates.
$M_{t,R}=4, M_{r,R}=8, M_{t,C}=8, M_{r,C}=4,$
$P_R=10LM_{t,R}, C=20$bits/symbol.

Simulations

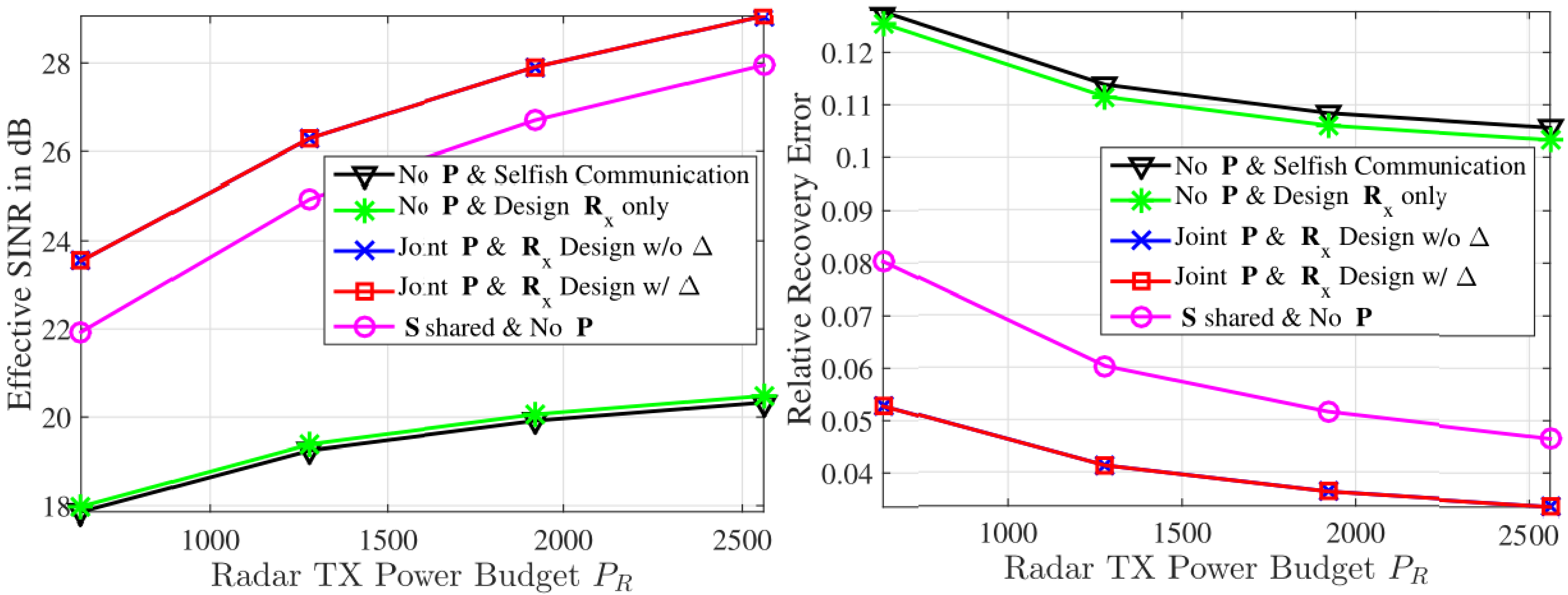


Figure 2. Spectrum sharing under different radar transmit power budget. $M_{t,R} = 4, M_{r,R} = 8, M_{t,C} = 8, M_{r,C} = 4,$ $p = 0.8, C = 20$bits/symbol.

Conclusions

- We have investigated a new framework for spectrum sharing between a MIMO communication system and a MIMO-MC radar system.
- Spectrum sharing is achieved by joint design of the radar precoding matrix and the communication codeword covariance matrix.
- Simulation results show that, the radar precoder plays a key role in improving the radar SINR and matrix completion recovery accuracy over previous approaches.
- Potential future directions include
  - spectrum sharing problem with targets distributed across different range bins
  - spectrum sharing in an environment with clutter.

Thank you!