# Efficient Target Estimation in Distributed MIMO Radar via the ADMM

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# Overview

- We consider the problem of target estimation in distributed MIMO radars that employ compressive sensing.
- We formulate a sparse signal recovery problem with
    - magnitude constraints on the target reflection coefficients;
    - a special structure for the signal to be recovered consisting of equal size blocks that have the same sparsity profile.
- A solution is proposed based on the alternating direction method of multipliers (ADMM), which
    - is computational more efficient as compared to algorithms based on the interior point method;
    - has improved estimation accuracy resulting from exploiting prior information on the target reflection coefficients;
    - is robust over a wide range of a manually chosen parameter.
- A parallel implementation and a decentralized scheme are discussed.

# Distributed MIMO Radar Using Sparse Signal Recovery

- We consider a MIMO radar system with $M_t$ transmit nodes (TX) and $M_r$ receive nodes (RX) that are widely separated.
- To exploit the spatial sparsity of the targets, the location space is discretized on the grid $\boldsymbol{\Theta} = \{(x_n, y_n), n = 1, \ldots, N\}$.
- The received baseband signal at the $j$-th RX $z_{ij}(t)$ arising due to the transmission of the $i$-th TX [Petropulu, Yu & Huang 2011]:

$$z_{ij}(t) = \sum_{n=1}^{N} s_{ij}^n x_i(t - \tau_{ij}^n) + n_{ij}(t) \quad (1)$$

| | |
|---|---|
| $x_i(t)$ | The $i$-th waveform |
| $s_{ij}^n$ | Reflectivity associated with the $n$-th grid point and TX/RX pair $(i,j)$ |
| $\tau_{ij}^n$ | Time delay associated with the $n$-th grid point and TX/RX pair $(i,j)$ |
| $n_{ij}(t)$ | Noise for TX/RX pair $(i,j)$ |

- If there is a target located on the $n$-th grid point, then $s_{ij}^n$ is the target complex RCS with $|s_{ij}^n| \in [0, \omega_0] \triangleq \boldsymbol{\Omega}$; otherwise $s_{ij}^n$ is zero.

# Distributed MIMO Radar Using Sparse Signal Recovery

- Obtain $L$ $T_s$-spaced samples and express in vector form

$$\mathbf{z}_{ij} = \mathbf{\Psi}_{ij}\mathbf{s}_{ij} + \mathbf{n}_{ij} \quad (2)$$

where $\mathbf{s}_{ij} = \left[s_{ij}^1, \ldots, s_{ij}^N\right]^T$ and

$$\mathbf{\Psi}_{ij} = \begin{bmatrix} x_i(t_0+0T_s-\tau_{ij}^1) & \cdots & x_i(t_0+0T_s-\tau_{ij}^N) \\ \vdots & \ddots & \vdots \\ x_i(t_0+(L-1)T_s-\tau_{ij}^1) & \cdots & x_i(t_0+(L-1)T_s-\tau_{ij}^N) \end{bmatrix}_{L\times N}$$

- The signal model for the overall MIMO radar system is

$$\mathbf{z} = \left[(\mathbf{z}_{11})^T, \ldots, (\mathbf{z}_{M_tM_r})^T\right]^T = \mathbf{\Psi}\mathbf{s} + \mathbf{n} \quad (3)$$

where $\mathbf{\Psi} = \text{diag}(\mathbf{\Psi}_{11}, \cdots, \mathbf{\Psi}_{M_tM_r})$ , $\mathbf{s} = \left[(\mathbf{s}_{11})^T, \ldots, (\mathbf{s}_{M_tM_r})^T\right]^T$ and $\mathbf{n} = \left[(\mathbf{n}_{11})^T, \ldots, (\mathbf{n}_{M_tM_r})^T\right]^T$.

- $\mathbf{s}$ exhibits group sparsity: it is composed by $M_tM_r$ sub-blocks, which share the same sparsity profile.

# Distributed MIMO Radar Using Sparse Signal Recovery

- Group sparsity (also known as block sparsity) was exploited to achieve improved target estimation and further reduction of the number of measurements needed.
- Existing block sparse recovery methods used for distributed MIMO radars include
  - Block Orthogonal Matching Pursuit (BOMP) [Gogineni, Nehorai 2011]: Poor perfomance in noise
  - Group Lasso with proximal gradient algorithm (GLasso) [Petropulu, Yu & Huang 2011]: High complexity, sensitive to the manually tuned parameter
  - mixed $\ell_1/\ell_2$ norm optimization (L-OPT) [Li, Petropulu 2014]: High compexity, assuming known noise variance
- We are aiming for a recovery method with
  - low complexity and robust performance
  - flexibility of incoporating prior information

# Fast Signal Recovery based on ADMM

- Reformulate for real variables:

$$\underbrace{\begin{bmatrix}\Re\{\mathbf{z}\}\\ \Im\{\mathbf{z}\}\end{bmatrix}}_{\tilde{\mathbf{z}}} = \underbrace{\begin{bmatrix}\mathbf{\Psi} & \\ & \mathbf{\Psi}\end{bmatrix}}_{\tilde{\mathbf{\Psi}}} \underbrace{\begin{bmatrix}\Re\{\mathbf{s}\}\\ \Im\{\mathbf{s}\}\end{bmatrix}}_{\tilde{\mathbf{s}}} + \underbrace{\begin{bmatrix}\Re\{\mathbf{n}\}\\ \Im\{\mathbf{n}\}\end{bmatrix}}_{\tilde{\mathbf{n}}} \quad (4)$$

where $\tilde{\mathbf{\Psi}}$ is still block diagonal, and $\tilde{\mathbf{s}} \in \mathbb{R}^{2NM_tM_r}$ has group sparsity.

- Solve the convex optimization problem

$$\min \frac{1}{2}\|\tilde{\mathbf{z}} - \tilde{\mathbf{\Psi}}\tilde{\mathbf{s}}\|_2^2 + \lambda \sum_{n=1}^{N} \|\tilde{\mathbf{s}}[\mathcal{I}_n]\|_2$$
$$s.t.\ \tilde{\mathbf{s}} \in \mathbf{\Omega}^{2NM_tM_r} \quad (5)$$

- The set $\mathcal{I}_n, \forall n \in \mathbb{N}_N^+$ with cardinality $2M_tM_r$ indexes out entries in $\tilde{\mathbf{s}}$ corresponding to the $n$-th grid point.
- The constraint $\tilde{\mathbf{s}} \in \mathbf{\Omega}^{2NM_tM_r}$ is satisfied if $\|\left[\tilde{\mathbf{s}}[i], \tilde{\mathbf{s}}[i+NM_tM_r]\right]\|_2 \in \mathbf{\Omega},\ \forall i \in \mathbb{N}_{NM_tM_r}^+$.

# Fast Signal Recovery based on ADMM

- In order to use Alternating Direction Method of Multipliers (ADMM), we introduce the auxillary variables $\mathbf{y}$ and $\mathbf{x}$. The problem then becomes

$$\begin{aligned} \min \frac{1}{2}\|\tilde{\mathbf{z}} - \tilde{\mathbf{\Psi}}\tilde{\mathbf{s}}\|_2^2 + \sum_{n=1}^{N} \lambda \|\mathbf{y}_n\|_2 \\ s.t.\ \mathbf{y}_n = \mathbf{D}_n \tilde{\mathbf{s}},\ \forall n \in \mathbb{N}_N^+; \\ \mathbf{x} = \tilde{\mathbf{s}}, \quad \mathbf{x} \in \mathbf{\Omega}^{2NM_tM_r} \end{aligned} \tag{6}$$

where the matrix $\mathbf{D}_n$ selects the entries of $\tilde{\mathbf{s}}$ indexed by $\mathcal{I}_n$. We have $\mathbf{y} = \mathbf{D}\tilde{\mathbf{s}}$ where $\mathbf{D} = [\mathbf{D}_1^T, \ldots, \mathbf{D}_N^T]$, $\mathbf{y} \triangleq [\mathbf{y}_1^T, \ldots, \mathbf{y}_N^T]^T$.

- $\mathbf{y}$ is a permutation of $\tilde{\mathbf{s}}$ and has block sparsity.
- The auxiliary variable $\mathbf{y}$ is used to isolate $\tilde{\mathbf{s}}$ from the group sparsity-inducing term $\sum \|\cdot\|_2$; the magnitude constraint is now imposed on $\mathbf{x}$ instead of $\tilde{\mathbf{s}}$.

# Fast Signal Recovery based on ADMM

- The augmented Lagrangian can be written as

$$\mathcal{L}(\tilde{\mathbf{s}}, \mathbf{y}, \mathbf{x}; \mu, \nu) = \frac{1}{2}\|\tilde{\mathbf{z}} - \tilde{\boldsymbol{\Psi}}\tilde{\mathbf{s}}\|_2^2 + \nu^T(\mathbf{x} - \tilde{\mathbf{s}}) + \frac{\rho_2}{2}\|\mathbf{x} - \tilde{\mathbf{s}}\|_2^2 \\ + \sum_{n=1}^{N}\left(\lambda\|\mathbf{y}_n\|_2 + \mu_n^T(\mathbf{y}_n - \mathbf{D}_n\tilde{\mathbf{s}}) + \frac{\rho_1}{2}\|\mathbf{y}_n - \mathbf{D}_n\tilde{\mathbf{s}}\|_2^2\right) \tag{7}$$

where $\rho_1, \rho_2 > 0$ and $\mu \triangleq [\mu_1^T, \ldots, \mu_N^T]^T \in \mathbb{R}^{2NM_tM_r}$ and $\nu \in \mathbb{R}^{2NM_tM_r}$ are the Lagrangian multipliers.

- ADMM is applicable if we group the variables into two blocks, i.e., $(\mathbf{y}, \mathbf{x})$ and $\tilde{\mathbf{s}}$.

$$(\mathbf{y}^{k+1}, \mathbf{x}^{k+1}) = \arg\min_{\mathbf{y},\mathbf{x}\in\boldsymbol{\Omega}^{\mathbf{NM_tM_r}}} \mathcal{L}(\tilde{\mathbf{s}}^k, \mathbf{y}, \mathbf{x}; \mu^k, \nu^k),$$

$$\tilde{\mathbf{s}}^{k+1} = \arg\min_{\tilde{\mathbf{s}}} \mathcal{L}(\tilde{\mathbf{s}}, \mathbf{y}^{k+1}, \mathbf{x}^{k+1}; \mu^k, \nu^k),$$

$$\nu^{k+1} = \nu^k + \rho_2(\mathbf{x}^{k+1} - \tilde{\mathbf{s}}^{k+1}),$$

$$\mu^{k+1} = \mu^k + \rho_1(\mathbf{y}^{k+1} - \mathbf{D}\tilde{\mathbf{s}}^{k+1}).$$

# ADMM iterations (1)

- The iterations for multipliers $\mu$ and $\nu$ are performed at cost $\mathcal{O}(NM_tM_r)$
- The $\mathbf{y}$-subproblem has computation cost $\mathcal{O}(NM_tM_r)$

$$\mathbf{y}_n^{k+1} = \max\left\{\|\bar{\mathbf{s}}_n^k\|_2 - \frac{\lambda}{\rho_1}, 0\right\} \frac{\bar{\mathbf{s}}_n^k}{\|\bar{\mathbf{s}}_n^k\|_2}, \ \forall n \in \mathbb{N}_N^+, \tag{8}$$

where $\bar{\mathbf{s}}_n^k = \mathbf{D}_n\tilde{\mathbf{s}}^k - \mu_n^k/\rho_1$ . Recall that multiplying by $\mathbf{D}_n$ only invovles index selection.

- The $\mathbf{x}$-subproblem has computation cost $\mathcal{O}(NM_tM_r)$

$$\mathbf{x}^{k+1} = \mathcal{P}_\Omega\left(\tilde{\mathbf{s}}^{k+1} - \frac{\nu^k}{\rho_2}\right), \tag{9}$$

where $\mathcal{P}_\Omega(\mathbf{x})$ projects $\left(\mathbf{x}[i], \mathbf{x}[i+NM_tM_r]\right)$ onto the region $\{(x, y)|x^2 + y^2 \leq \omega_0\}$ for all $i \in \mathbb{N}_{NM_tM_r}^+$.

# ADMM iterations (2)

- For the $\tilde{\mathbf{s}}$-subproblem, the minimum is achieved by

$$0 \in \frac{\partial}{\partial \tilde{\mathbf{s}}} \mathcal{L}(\tilde{\mathbf{s}}, \mathbf{y}^{k+1}, \mathbf{x}^{k+1}; \mu^k, \nu^k) = \mathbf{A}\tilde{\mathbf{s}} - \mathbf{b}^k \quad (10)$$

where $\mathbf{A} = \tilde{\boldsymbol{\Psi}}^T \tilde{\boldsymbol{\Psi}} + (\rho_1 + \rho_2)\mathbf{I}_{2NM_tM_r}$ is block-diagonal and fixed in each iteration; $\mathbf{b}^k = \tilde{\boldsymbol{\Psi}}^T \tilde{\mathbf{z}} + \mathbf{D}^T \mu^k + \rho_1 \mathbf{D}^T \mathbf{y}^{k+1} + \nu^k + \rho_2 \mathbf{x}^{k+1}$.

- System (10) can be decomposed into a set of subsystems of equations, i.e.,

$$\mathbf{A}_m \tilde{\mathbf{s}}_m^{k+1} = \mathbf{b}_m^k, \ \forall m \in \mathbb{N}^+_{2M_tM_r}, \quad (11)$$

where $\mathbf{A}_m = \begin{cases} \boldsymbol{\Psi}_{ij}^T \boldsymbol{\Psi}_{ij} + (\rho_1 + \rho_2)\mathbf{I}_N & \text{if } m \in [1, M_tM_r] \\ \mathbf{A}_{m-M_tM_r} & \text{otherwise} \end{cases}$

with $j = \lfloor \frac{m-1}{M_t} \rfloor + 1$ and $i = m - (j-1)M_t$.

- $\mathbf{A}_m$ is guaranteed to be strictly diagonal dominant and symmetric. The total number of operations to solve (11) is $\mathcal{O}(N^2 M_t M_r)$.

# Convergence and Advantages

- The convergence of the above ADMM iterations is guaranteed by results in the ADMM literature.
- The computational cost is low: $\mathcal{O}(N^2 M_t M_r)$ v.s. $\mathcal{O}((NM_t M_r)^3)$ for interior point based methods .
- The estimation accuracy is improved by introducing the amplitude constraints.
- The performance is robust over wide range of regularization parameter $\lambda$ (verified by the simulations).
- The iterations of all variables exhibit separability.

# Implementation Schemes and Discussions

- Parallel Implementation
  - all pairs $(\mathbf{x}^k[i], \mathbf{x}^k[i + NM_tM_r])$ in $\mathbf{x}^k$ are updated independent of others
  - a similar parallel scheme applies to $\mu^k$ and $\nu^k$, and the update of $\mathbf{y}_n^k$.
- Fusion Center Aided Semi-Distributed Implementation
  - $\mathbf{x}$, $\mathbf{s}$ and $\nu$ are divided into blocks, each of which can be updated locally at one receive node;
    The receive node $j$ updates $\mathbf{x}_m^{k+1}, \nu_m^{k+1}$ and $\mathbf{s}_m^{k+1}$ for all $m \in \mathcal{T}_j \triangleq \{(j-1)M_t + i, M_tM_r + (j-1)M_t + i | i \in \mathbb{N}_{M_t}^+\}$. The computation cost is $\mathcal{O}(N^2 M_t)$ at each node. $v_m^{k+1} \in \mathbb{R}^N$ , $m \in \mathbb{N}_{2M_tM_r}^+$, denotes the $m$-th block of the uniformly partitioned vector $v^{k+1}$.
  - A fusion center performs the update of $\mathbf{y}$ and $\mu$;
    The computation cost is $\mathcal{O}(NM_tM_r)$ at the fusion center.
  - In each iteration, each receive node uploads $\tilde{\mathbf{s}}_m^{k+1}$ and downloads $\mathbf{y}_m^{k+1}$ from the fusion center.
    $\mathbf{y}_m^{k+1} \in \mathbb{R}^N$denotes the $m$-th block of the uniformly partitioned $\mathbf{D}^T\mathbf{y}^{k+1}$.

# Simulations (1)

- We evaluate the performance of our proposed method using as metrics estimation error and running time.

Simulation setup

- $4$ transmit and $4$ receive nodes, waveforms with joint Gaussian entries;
- SNR $= 5dB$;
- $25 \times 10$ grid points with $10m$ grid size;
- The magnitude of the complex reflection coefficients has uniform distribution $\mathcal{U}[0.1, 0.8]$. $\omega_0$ is chosen as $1$.

Comparison methods

- BOMP [Gogineni, Nehorai 2011] ;
- GLasso using proximal gradient methods [Petropulu, Yu & Huang 2011];
- L-OPT with $\epsilon = 2\sqrt{LM_tM_r}\sigma_n$ [Li, Petropulu 2011] with knowledge of $\sigma_n$.

# Simulations (2)

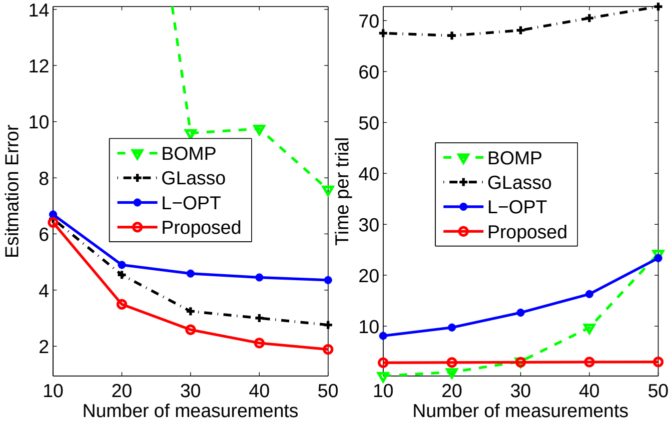


Figure: Performance under different number of measurements. $10$ targets; for GLasso $\lambda = 0.02$; and for the proposed method $\lambda = 2$, $\rho_1 = \rho_2 = 1$.

# Simulations (3)

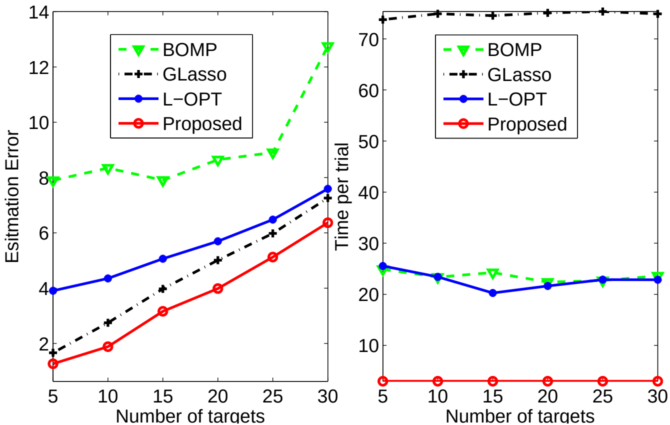


Figure: Performance under different number of targets. $50$ measurements; for GLasso $\lambda = 0.02$; and for the proposed method $\lambda = 2$, $\rho_1 = \rho_2 = 1$.

# Simulations (4)

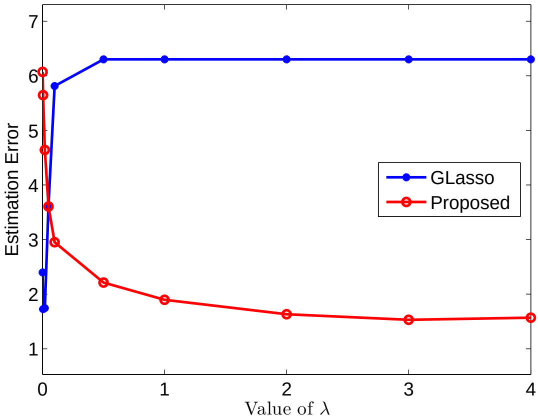


Figure: Performance under different values of $\lambda$.

# Conclusions

- An ADMM-based efficient sparse signal recovery algorithm has been proposed for target estimation in distributed MIMO radar.
- Simulation results have indicated that the proposed algorithm significantly lowers the computational complexity for target estimation and improves accuracy.
- Parallel implementation has also been considered for further reduction of the execution time. A semi-distributed implementation, requiring a fusion enter with minimal computational power, has also been discussed.

# Thank You

# References I

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